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editorial

Let us Teach Popular Mathematics

What we have to take up as an urgent reformation in education is to develop methods of learning Mathematics with a smile. Wherever there is quantitative reasoning, Mathematics has to be applied.

The borders of Mathematics are expanding. They have to be applied to every branch of Physics, Chemistry, Biology, Archaeology, analytical methods using spectrometers of every type and every industrial technology. We have a feeling that as most of the good spectrometers ranging from optical, X-ray, mass spectrometry, X-ray and electron diffraction instruments have software fitted to give ready results and therefore they are very accurate.

By developing one's own methods, one can make a revolution in the methods of analysis. Learning Mathematics can be exciting when one starts going deeper to understand the basic reasoning as well as assumptions. For analysis, the most powerful technique is statistics. Yes, we all know that learning Mathematics is very important. Yes, we also know that students who are very brilliant in many fields also dislike Mathematics. The only obstacle is the presentation. Even when choosing candies, children pick up the brightly packed ones. This is human nature.

The need of the hour is to stop the students running away from science classes. The easiest way to popularise Mathematics is to start from concrete examples to generalisation. There are Mathematics books which read like novels. We have to teach the students how to draw mathematical conclusions from the concrete concepts. Statistics is one such subject used by all streams. Our suggestion is to take route from applications to theory at least at the school stage.

Anil Ahlawat
Editor

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Maths Musing was started in January 2003 issue of Mathematics Today with the suggestion of Shri Mahabir Singh. The aim of Maths Musing is to augment the chances of bright students seeking admission into IITs with additional study material.

During the last 10 years there have been several changes in JEE pattern. To suit these changes Maths Musing also adopted the new pattern by changing the style of problems. Some of the Maths Musing problems have been adapted in JEE benefitting thousand of our readers. It is heartening that we receive solutions of Maths Musing problems from all over India.

Maths Musing has been receiving tremendous response from candidates preparing for JEE and teachers coaching them. We do hope that students will continue to use Maths Musing to boost up their ranks in JEE Main and Advanced.

Prof. Dr. Ramanaiah Gundala
Former Dean of Science and Humanities, Anna University, Chennai

PROBLEM Set 134

JEE MAIN

1. Let $OABC$ be a tetrahedron whose edges are of unit length. If $\overrightarrow{OA} = \vec{a}$, $\overrightarrow{OB} = \vec{b}$ and $\overrightarrow{OC} = \alpha(\vec{a} + \vec{b}) + \beta\vec{a} \times \vec{b}$, then $\frac{\beta}{\alpha} =$

(a) $\frac{1}{\sqrt{2}}$ (b) $\sqrt{2}$ (c) 2 (d) $2\sqrt{2}$

2. A man throws a fair coin a number of times and gets 2 points for each head the throws and 1 point for each tail. The probability that he gets exactly 6 points is

(a) $\frac{21}{32}$ (b) $\frac{23}{32}$ (c) $\frac{41}{64}$ (d) $\frac{43}{64}$

3. If $f(\pi) = 2$ and $\int_0^{\pi} (f(x) + f''(x)) \sin x dx = 5$, then $f(0) =$

(a) 3 (b) 2 (c) 1 (d) 0

4. The sum of the moduli of all the roots of $x^4 - 3x^3 - 2x^2 - 3x + 1 = 0$ is

(a) 2 (b) 4 (c) 6 (d) 8

5. $ABCD$ is a cyclic quadrilateral of area $4\sqrt{3}$. If $AB = 2$, $BC = 5$ and $\angle B = \frac{\pi}{3}$, then its perimeter is

(a) 14 (b) 12 (c) 10 (d) 8

JEE ADVANCED

6. If the points (2, 0), (3, 2), (5, 4) and (t, 0) are concyclic then $t =$

(a) 1 (b) 2 (c) 17 (d) 19

COMPREHENSION

In triangle ABC, h_1, h_2, h_3 are the lengths of altitudes from A, B, C. The altitudes are natural numbers forming an increasing H.P. If $h_2 = 16$, then

7. The radius $r =$

(a) $\frac{15}{4}$ (b) $\frac{16}{3}$ (c) $\frac{15}{2}$ (d) $\frac{16}{5}$

8. The circumradius $R =$

(a) $\frac{128}{5}$ (b) $\frac{256}{3}$ (c) $\frac{256}{5}$ (d) $\frac{256}{15}$

INTEGER MATCH

9. Let n be the number of 5-letter words using the letters of the word VARANASI. The sum of the digits of n is

MATCHING LIST

10. List-I contains S and List-II gives last digit of S.

List-I		List-II	
P.	$S = \sum_{n=1}^{11} (2n-1)^2$	1.	0
Q.	$S = \sum_{n=1}^{10} (2n-1)^3$	2.	1
R.	$S = \sum_{n=1}^{18} (2n-1)^2(-1)^n$	3.	5
S.	$S = \sum_{n=1}^{15} (2n-1)^3(-1)^{n-1}$	4.	8

	P	Q	R	S
(a)	4	2	1	3
(b)	4	2	3	1
(c)	2	1	4	3
(d)	1	2	3	4

See Solution set of Maths Musing 133 on page no. 86

Prof. Ramanaiah is the author of MTG JEE(Main & Advanced) Mathematics series

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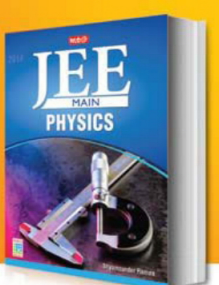

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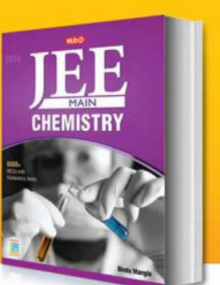

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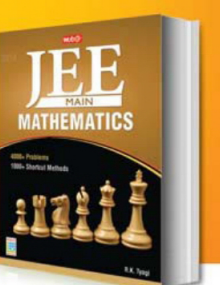

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PRACTICE PAPER 2014

JEE Main & Advanced

Topic : Permutations and Combinations

SECTION-I

Single Correct Answer Type

1. The number of divisors of 3630, which have a remainder of 1 when divided by 4, is
(a) 12 (b) 6
(c) 4 (d) none of these

2. The number of zeros at the end of 127! is
(a) 28 (b) 29 (c) 30 (d) 31

3. Five digit number divisible by 3 is formed using 0, 1, 3, 4, 6, 7 without repetition. Total number of such numbers are
(a) 312 (b) 3125 (c) 120 (d) 7216

4. India and West Indies play one-day international cricket series until one team wins 4 matches. No match ends in a draw. Number of ways, the series can be won by India is
(a) 70 (b) 120 (c) $\frac{5!}{2}$ (d) 35

5. If $\frac{{}^{n}P_{r-1}}{a} = \frac{{}^{n}P_{r}}{b} = \frac{{}^{n}P_{r+1}}{c}$ then
(a) $b^2 = a(b + c)$ (b) $\frac{1}{a} + \frac{1}{b} + \frac{1}{c} = 1$
(c) $a^2 = c(a + b)$ (d) $abc = 1$

6. A class contains three girls and four boys. Every saturday five students go on a picnic, a different group being sent each week. During the picnic, each girl in the group is given a doll by the accompanying teacher. After all possible groups of the five have gone once, the total no. of dolls the girls have got is
(a) 27 (b) 11 (c) 21 (d) 45

7. Three ladies, each brought a child for admission to a school. The head of the school wishes to interview six people one by one, taking case that no child is interviewed before its mother. The no. of different ways, this can be done is
(a) 6! (b) 3! × 3! × 2
(c) $\frac{6!}{3!3!}$ (d) 90

8. Triplet (x, y, z) is chosen from the set $\{1, 2, 3, \ldots, n\}$, such that $x \le y < z$. The number of such triplets is
(a) n^3 (b) ${}^{n}C_3$ (c) ${}^{n}C_2$ (d) ${}^{n}C_2 + {}^{n}C_3$

9. m points on one straight line are joined to n points on another straight line. The number of point of intersection of the line segments thus formed is
(a) ${}^{m}C_2 \div {}^{n}C_2$ (b) $\frac{mn(m-1)(n-1)}{4}$
(c) $\frac{{}^{m}C_2 \cdot {}^{n}C_2}{2}$ (d) ${}^{m}C_2 + {}^{n}C_2$

10. There are 8 lamps in a hall. Each one of them can be switched on independently. The number of ways in which the hall can be illuminated is
(a) 8! (b) 16 (c) 255 (d) 2^8

SECTION-II

Mutiple Correct Answer Type

11. If $10! = 2^p \cdot 3^q \cdot 5^r \cdot 7^s$, then
(a) $2q = p$
(b) $pqrs = 64$
(c) 10! is divisible by 280
(d) the product of two integers in 10! is 135, when 10! is arranged in a row.

12. If $P = 21(21^2 - 1^2)(21^2 - 2^2)(21^2 - 3^2) \ldots (21^2 - 10^2)$, then P is always divisible by
(a) 22! (b) 21! (c) 19! (d) 20!

13. The number of ways in which 30 identical things can be distributed among 6 people is
(a) ${}^{17}C_5$, when each people gets odd number of things
(b) ${}^{16}C_{11}$, when each people gets odd number of things
(c) ${}^{14}C_5$, when each people gets even number of things

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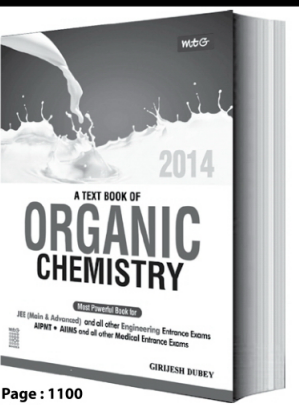

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(d) $^{15}C_{10}$, when each people gets even number of things.

14. If n number of straight lines satisfy the equation

$$^{(n+5)}P_{(n+1)} = \frac{11(n-1)}{2} \cdot {}^{(n+3)}P_{n}.$$

Then the number of point of intersections of the straight lines will be

(a) 15 (b) 28 (c) 21 (d) 10

15. The number of ways, in which 100 husband-wife out of 200 people, can be arranged are

(a) $\frac{(200)!}{2^{100}\cdot(100)!}$

(b) $1 \times 3 \times 5 \times \ldots \times 199$

(c) $\left(\frac{101}{2}\right)\left(\frac{102}{2}\right)\ldots\left(\frac{200}{2}\right)$

(d) $\frac{(200)!}{(100)!}$

SECTION-III

Assertion-Reason Type

Direction : (16-20) : Each question contains two statements. Statement-1 (Assertion) and Statement-2 (Reason). Each of the following questions also has four alternative choices, only one of which is correct answer. You have to select the correct choice.

(a) Both statement-1 and statement-2 are true, statement-2 is a correct explanation of statement-1

(b) Statement-1 is true and statement-2 is false

(c) Statement-1 is false and statement-2 is true

(d) Both Statement-1 and statement-2 are true, but statement-2 is not a correct explanation of statement-1

16. **Statement-1 :** There are 5 integers between 1 and 200, each of which has ten divisors.
Statement-2 : If p_1, p_2, p_3 are distinct primes, then the number of divisors of $p_1^{\alpha_1} \times p_2^{\alpha_2} \times p_3^{\alpha_3}$ is $(1+\alpha_1)(1+\alpha_2)(1+\alpha_3)$.

17. In a shop there are five types of ice-creams available. A child buys six ice-creams.
Statement-1 : The number of different ways the child can buy the six ice-creams is $^{10}C_5$.
Statement-2 : The number of different ways the child can buy the six ice-creams is equal to the number of different ways of arranging 6A's and 4 B's in a row.

18. **Statement-1 :** If a set of 10 boys are divided into three subsets of 3, 3, 4 boys, the probability that the oldest boy is in a subset of 3 members and the youngest boy is in the subset of 4 members is $\frac{4}{15}$.
Statement-2 : The number of ways of forming three subsets of m, m, n members from a set of $(2m+n)$ members is $\frac{(2m+n)!}{2(m!)^2 n!}$.

19. **Statement-1 :** If a polygon has 66 diagonals then number of its sides is 12.
Statement-2 : Number of ways of selecting 2 points out of n-non collinear points is $^nC_2$.

20. **Statement-1 :** The number of selections of four letter word taken from the word PARALLEL must be 15.
Statement-2 : Co-efficient of x^4 in $(1-x)^{-3}$ is 15.

SECTION-IV

Comprehension Type

Paragraph for Q. No. 21 to 23

In a cricket team there are 8 batsmen, 6 bowlers and 2 wicket-keepers. 4 are specialist in both bat and bowl. A team of 11 players is to be formed.

21. The number of teams that can be formed, where there will be always a wicket-keeper and an all rounder are

(a) $^4C_1 \times {}^{14}C_{10} + {}^2C_1 \times {}^{14}C_{10} + {}^4C_1 \times {}^2C_1 \times {}^{14}C_9 + {}^{14}C_{11}$

(b) $^4C_1 \times {}^{15}C_{10} + {}^{15}C_{11}$

(c) $^4C_1 \times {}^{15}C_{10} - {}^{15}C_{11}$

(d) none of these

22. When a particular bowler is in the team, then two particular batsman refuse to play in that team. In such a situation the number of teams that can be formed with those players are

(a) $^{17}C_{10} + {}^{19}C_{11}$ (b) $^{17}C_{10} + {}^{19}C_{11} + {}^{17}C_{11}$

(c) $^{17}C_{10} + {}^{20}C_{11}$ (d) $^{19}C_{10} + {}^{19}C_{11}$

23. The number of teams that can be formed, when a particular batsman is in the team, then a particular player refuses to play in that team, are

(a) $2 \cdot {}^{18}C_{10}$ (b) $^{19}C_{11} + {}^{18}C_{10}$

(c) $^{17}C_{10} + {}^{19}C_{11}$ (d) none of these

Paragraph for Q. No. 24 to 26

The word MATHEMATICS consists of 11 letters.

24. The number of arrangements of the letters of the word, where the odd places contain at least one repeated letters are

(a) $\frac{11!}{2!2!2!} - \frac{9!}{2!2!}$ (b) $\frac{9!}{2!2!2!}$

(c) $\frac{9!}{2!2!}$ (d) $\frac{11!}{2!2!2!}$

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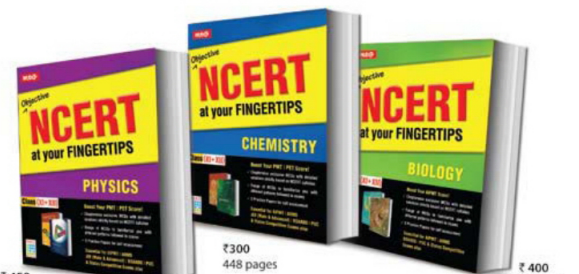

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25. The number of permutations, where two M's and two T's are together, but two A's are not together, are

(a) $\frac{11!}{2!2!2!} - \frac{10!}{2!2!}$ (b) $7! \, {}^8C_2$

(c) $\frac{6!4!}{2!2!}$ (d) $\frac{9!}{2!2!2!}$

26. Two vowels are never together such type of permutations are

(a) $7! \, {}^8C_4 \cdot \frac{4!}{2!}$ (b) $\frac{7!}{2!} \cdot {}^8C_4 \cdot \frac{4!}{2!}$

(c) $\frac{7!}{2!2!} \cdot {}^8C_4 \cdot \frac{4!}{2!}$ (d) $\frac{7!}{2!2!2!} \cdot {}^8C_4 \cdot \frac{4!}{2!}$

SECTION-V

Matrix Match Type

27. Match the following:
Consider all possible permutations of the letters of the word ENDEANOEL.

	Column-I	Column-II	
(A)	The number of permutations of the word ENDEA is	(p)	5!
(B)	The number of permutations in which the letter E occurs in the first and the last position is	(q)	2 × 5!
(C)	The number of permutations in which none of the letters D, L, N occurs in the last five positions is	(r)	7 × 5!
(D)	The number of permutations in which the letters A, E, O occurs only in odd positions is	(s)	21 × 5!

28. Match the following:

	Column-I	Column-II	
(A)	The number of ways of forming 2 teams from 5 boys and 5 girls, so that each team has 5 children and girls are in majority in a team is	(p)	127
(B)	The number of ways, in which 12 red balls, 12 black balls and 12 white balls be given to 2 children each 18, is	(q)	125
(C)	Six bundles of books are to be kept in 6 boxes one in each box. If 2 of the boxes are too small for three of the bundles, the number of ways of keeping the bundles in the boxes is	(r)	135
(D)	A bag contains 6 black, 6 blue, 6 red, 6 green and 6 white balls. The balls are numbered 1 to 6 in each colour. The number of ways of drawing 2 balls from the bag such that the balls are of the same colour or the numbers on them are same, is	(s)	144

SECTION-VI

Integer Answer Type

29. In a class of 10 students, there are 3 girls. The number of ways they can be arranged in a row, so that no two girls are consecutive, is K · 8! where the sum of the digits of K is

30. If the letters of the word SACHIN are arranged in all possible ways and these words are written in dictionary order, then the word SACHIN appears at a serial number *K*. The sum of the digits of *K* is

31. A guard of 12 men is formed from a group of *n* soldiers in all possible ways. If the number of times two particular soldiers A and B are together on guard is thrice the number of times three particular soldiers C, D, E are together on guard, then the sum of the digits of *n* is

32. A father with 8 children, takes 3 at a time to zoo as often as he can without taking the same children together more than once. The number of times he visits zoo is *x* and the number of times each child visits zoo is *y*. Then unit place digit is

SOLUTIONS

1. (b): $3630 = 2 \times 3 \times 5 \times 11^2$

odd divisors of 3630 are terms of the product

$(1 + 3)(1 + 5)(1 + 11 + 11^2)$

i.e., $(1 + 3 + 5 + 15)(1 + 11 + 121)$

i.e., $(1 + 3 + 5 + 15 + 11 + 33 + 55 + 165 + 121 + 363 + 605 + 1905)$

$\therefore$ Divisors of 3630 which have remainder 1 when divided by 4 are 1, 5, 165, 121, 605, 1905.

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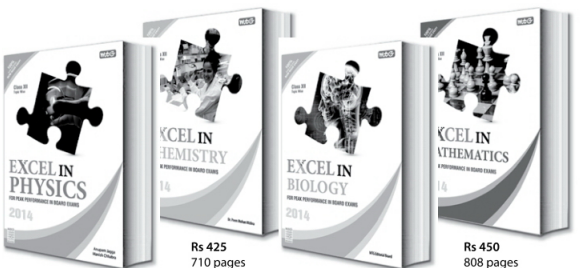

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2. (d): For the number of zeros in any factorial number, we always find $E_2(n!)$ and $E_5(n!)$

$$\therefore \quad E_2(127!) = \left[\frac{127}{2}\right] + \left[\frac{127}{4}\right] + \left[\frac{127}{8}\right] + \left[\frac{127}{16}\right] + \left[\frac{127}{32}\right] + \left[\frac{127}{64}\right]$$

$$= 63 + 31 + 15 + 7 + 3 + 1$$

$$= 120$$

$$E_5(127!) = \left[\frac{127}{5}\right] + \left[\frac{127}{25}\right] + \left[\frac{127}{125}\right]$$

$$= 25 + 5 + 1$$

$$= 31$$

$\therefore$ Number of zeros in 127! is minimum of (120, 31) = 31

3. (a): A number is divisible by 3 if sum of its digit is divisible by 3.

Now selection of 5 digits for forming five digit numbers divisible by 3 are

(i) 1, 3, 4, 6, 7 (ii) 0, 1, 4, 6, 7 and (iii) 0, 1, 3, 4, 7

In case (i) *i.e.*, when 1, 3, 4, 6, 7 are taken as digits, the number of 5 digit numbers will be 5! = 120.

In case (ii) and (iii) we have 4 × 4! in each case.

$\therefore$ Total number of required numbers

$$= 120 + 2 \times 96 = 312$$

4. (d): Taking I for India and W for West Indies, one of the way India can win the series is WIIII. If India wins the series, last match is always won by India. The following four cases are possible for India to win the series:

	INDIA	WESTINDIES
(i)	IIII	–
(ii)	IIII	W
(iii)	IIII	WW
(iv)	IIII	WWW

Clearly the number of ways in

Case (i) 1 = 1 way

Case (ii) $\frac{4!}{3!}$ = 4 ways

Case (iii) $\frac{5!}{2!3!}$ = 10 ways

Case (iv) $\frac{6!}{3!3!}$ = 20 ways

Hence total number of ways = 1 + 4 + 10 + 20 = 35

5. (a): $\frac{{}^nP_{r-1}}{a} = \frac{{}^nP_r}{b} = \frac{{}^nP_{r+1}}{c}$

$$\Rightarrow \quad \frac{1}{a} \cdot \frac{\lfloor n}{\lfloor n-r+1} = \frac{1}{b} \cdot \frac{\lfloor n}{\lfloor n-r} = \frac{1}{c} \cdot \frac{\lfloor n}{\lfloor n-r-1}$$

$$\Rightarrow \quad \frac{1}{a} \cdot \frac{\lfloor n}{(n-r+1)(n-r)} = \frac{1}{b} \cdot \frac{\lfloor n}{(n-r)} = \frac{1}{c} \cdot \frac{\lfloor n}{1}$$

$$\Rightarrow \quad \frac{1}{a(n-r+1)(n-r)} = \frac{1}{b} \cdot \frac{1}{n-r} = \frac{1}{c}$$

$$\Rightarrow \quad \frac{b}{a} = n - r + 1 \qquad \ldots(i)$$

and $$\frac{c}{b} = n - r \qquad \ldots (ii)$$

Putting value of (ii) in (i)

$$\therefore \quad \frac{b}{a} = \frac{c}{b} + 1 = \frac{b+c}{b}$$

$$\Rightarrow \quad b^2 = a(b + c)$$

6. (d): Out of 3 girls and 4 boys, we can have the following no. of different groups for picnic.

Case I : 2 boys + 3 girls

The number of ways we can have such group

$$= {}^4C_2 \times {}^3C_3 = 6 \text{ ways}$$

Hence 6 × 3 = 18 dolls will be given

Case II : 3 boys + 2 girls

No. of ways = ${}^4C_3 \times {}^3C_2 = 12$ ways.

Hence 12 × 2 = 24 dolls

Case III : 4 boys + 1 girl

No. of ways = ${}^4C_4 \times {}^3C_1 = 3$ ways

Hence 3 dolls.

$\therefore$ Total number of dolls = 18 + 24 + 3 = 45.

7. (d): Let the mothers be *A*, *B* and *D* and their respective children *a*, *b*, *d*.

Case I : When each mother is followed by her child *i.e. Aa*, *Bb*, *Dd*. This can be done in 3! *i.e.*, 6 ways.

Case II : *AB ab Dd* interview in this order may be conducted in 2! × 2! × ${}^3C_2$ ways *i.e.* 12 ways.

Similarly order of type

Dd AB ab again in 12 ways.

Case III : *AB aD bd* this order of interview can be conducted in

2! × 2! × 2! × ${}^3C_2$ = 24 ways

Case IV : *ABD abd* this order of interview, can be conducted in 3! × 3! ways = 36 ways.

$\therefore$ Total number of ways = 90.

8. (d): Number of selections when $x < y < z = {}^nC_3$

Number of selections when $x = y < z = {}^nC_2$

(Here in fact we have to select only two numbers out of *n* numbers)

$\therefore$ Required number = ${}^nC_3 + {}^nC_2$

9. (b): Required number of point of intersections

$$= {}^mC_2 \times {}^nC_2$$

$$= \frac{mn(m-1)(n-1)}{4}$$

10. (c): Each of the 8 switches can be handled in 2 ways *i.e.*, either switched on or switched off. Hence total number of ways of handling the 8 switches is

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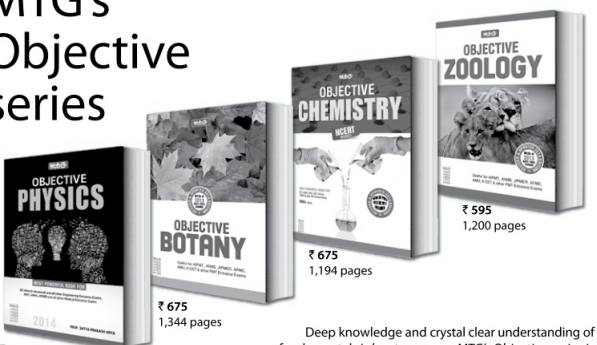

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2^8 which includes all the 8 lamps are off. Hence the number of ways of illuminating the hall is $2^8 - 1$.

11. (a, b, c, d) : $10! = 2 \times 3 \times 4 \times 5 \times 6 \times 7 \times 8 \times 9 \times 10$
$= (2 \times 4 \times 6 \times 8 \times 10) \times 3 \times 5 \times 7 \times 9$
$= 2^5(1 \times 2 \times 3 \times 4 \times 5) \times 3 \times 5 \times 7 \times 9$
$= 2^8 \times 3^4 \times 5^2 \times 7^1$

$\therefore \quad p = 8, q = 4, r = 2, s = 1$

$\therefore \quad p = 8 = 2 \times 4 = 2q$, thus option (a) is correct.

Now $pqrs = 8 \times 4 \times 2 \times 1 = 64$, option (b) is correct.

Again $10! = 2^8 \times 3^4 \times 5^2 \times 7$
$= 2^5 \times (2^2 \times 7 \times 2 \times 5) \times 3^4 \times 5$
$= 2^5 \times 280 \times 3^4 \times 5$

$\therefore$ 10! is divisible by 280. Thus option (c) is correct.

The product of two integers in 10! is 135 ($15 \times 9 = 135$). Thus option (d) is correct.

12. (b, d): $P = 21(21^2 - 1^2)(21^2 - 2^2)(21^2 - 3^2)\ldots(21^2 - 10^2)$
$= 21(21 + 1)(21 - 1)(21 + 2)(21 - 2)$
$(21 + 3)(21 - 3)\ldots(21 + 10)(21 - 10)$
$= 11 \times 12 \times 13 \ldots \times 21 \times 22 \times \ldots 31$
= Product of 21 consecutive integers.

$\therefore$ P is divisible by 21! and so by 20!

13. (a, c)

14. (a, c) : $^{(n+5)}P_{n+1} = \dfrac{11(n-1)}{2} \cdot {}^{(n+3)}P_n$

$\Rightarrow \quad \dfrac{\lfloor n+5}{\lfloor 4} = \dfrac{11(n-1)}{2} \times \dfrac{\lfloor n+3}{\lfloor 3}$

$\Rightarrow \quad \dfrac{(n+5)(n+4)}{24} = \dfrac{11(n-1)}{2 \times 6}$

$\Rightarrow \quad n^2 + 9n + 20 = 22(n-1)$

$\Rightarrow \quad n^2 - 13n + 42 = 0$

$\Rightarrow \quad (n-6)(n-7) = 0 \qquad \therefore \qquad n = 6, 7$

$\therefore$ The number of straight lines are either 6 or 7.

$\therefore$ The number of point of intersections are either $^6C_2 = 15$ or $^7C_2 = 21$.

15. (a, b, c)

16. (a) : The number of divisors is $10 = 2 \times 5$

$\therefore$ The numbers are of the form,

p^4 or pq^4 where p, q are primes.

The desired numbers are
$2^4 \cdot 3 = 48, 2^4 \cdot 5 = 80, 2^4 \cdot 7 = 112$
$2^4 \cdot 11 = 176, 3^4 \cdot 2 = 162$

17. (c) : The number of non-negative integer solutions of $x_1 + x_2 + \ldots + x_6 = \begin{pmatrix} 6+5-1 \\ 5-1 \end{pmatrix} = \begin{pmatrix} 10 \\ 4 \end{pmatrix}$

It is same as $\dfrac{(6+4)!}{6!4!} = \begin{pmatrix} 10 \\ 4 \end{pmatrix}$

18. (a) : The number of ways of forming three subsets of 3, 3, 4 members from a set of 10 members is $D = \dfrac{10!}{2(3!)^2 4!}$.

The number of ways of forming three subsets of 3, 3, 2 members from the set of 8 members obtained by removing the oldest and youngest boy is $E = \dfrac{8!}{2(3!)^2 2!}$

$\therefore$ The desired probability $= \dfrac{2E}{D} = \dfrac{4}{15}$

19. (c) : Number of diagonals of a n-sided polygon $= {}^nC_2 - n$

$= {}^{12}C_2 - 12 = 66 - 12 = 54$

20. (c) : The word 'PARALLEL' consists $L = 3$
$A = 2, P = 1, R = 1$ and $E = 1$

$\therefore$ Number of selection of 4 letters
= co-efficient of x^4 in $(1 + x + x^2 + x^3)(1 + x + x^2)(1 + x)$

= co-efficient of x^4 in $\dfrac{1-x^4}{1-x} \times \dfrac{1-x^3}{1-x} \times \dfrac{1-x^2}{1-x}$

= co-efficient of x^4 in $(1 - x^2 - x^3 - x^4 + x^5 + x^6 + x^7 - x^9)$
$\times (1 + 3x + 6x^2 + 10x^3 + 15x^4 + \ldots)$
$= 15 - 6 - 3 - 1 = 5$

$\therefore$ Statement 1 is false but statement 2 is true as co-efficient of x^4 in $(1 - x)^{-3}$ is $^6C_4 = {}^6C_2 = 15$

21. (a) : $^4C_1 \times {}^{14}C_{10} + {}^2C_1 \times {}^{14}C_{10} + {}^4C_1$
$\times {}^2C_1 \times {}^{14}C_9 + {}^{14}C_{11}$

22. (b): $^{17}C_{10} + {}^{19}C_{11} + {}^{17}C_{11}$

23. (b): $^{19}C_{11} + {}^{18}C_{10}$

24. (d): $\dfrac{11!}{2!2!2!}$

25. (b): $7! \cdot {}^8C_2$

26. (c) : $\dfrac{7!}{2!2!} \cdot {}^8C_4 \cdot \dfrac{4!}{2!}$

27. A → p; B → s; C → q; D → q

Considering (ENDEA) as a single letter, the word (ENDEA) NOEL can be arranged in $\lfloor 5 = 120$ ways

Keeping the first and last place for E, remaining letter of word ENDEANOEL can be arranged E(NDEANOL)

E in $\dfrac{\lfloor 7}{\lfloor 2} = 21 \times 5!$

According to the questions, the arrangements will be like

				E	E	A	O	E	
*	*	*	*	*	*	*	*	*	*
D	L	N	N						

$\therefore$ number of arrangements $= \frac{4!}{2!} \times \frac{5!}{3!} = 2 \times 5!$

There are 5 odd places in the word ENDEANOEL, and A, E, E, O can be arranged in those five places in $\frac{|5}{|3}$ ways and the remaining 4 letters N, D, N, L can be arranged in $\frac{|4}{|2}$ ways.

$\therefore$ Total number of arrangements $= \frac{5!}{3!} \times \frac{4!}{2!} = 2 \times 5!$

28. $A \to q$; $B \to p$; $C \to s$; $D \to r$

The 10 children can be made into 2 teams in $\frac{1}{2}\binom{10}{5}$ = 126 ways. Only one of these will have 5 girls. The desired number is 126 – 1 = 125

Coeffecient of x^{18} in $(1 + x + \ldots + x^{12})^3$

$= (1 - x^{13})^3 (1 - x)^{-3}$

$= (1 - 3x^{13})\left(1 + \binom{3}{1}x + \binom{4}{2}x^2 + \ldots\right)$

which is $\binom{20}{18} - 3\binom{7}{5} = \binom{20}{2} - 3\binom{7}{2} = 190 - 63 = 127$.

We first fill the small boxes with 2 of the three bundles and remaining boxes with the remaining bundles.

The required number is ${}^3P_2 \cdot 4! = 6 \times 24 = 144$.

We choose any colour and take 2 balls or take any number from 1 to 6 and choose 2 colours.

$\binom{5}{1}\binom{6}{2} + \binom{6}{1}\binom{5}{2} = 75 + 60 = 135$

29. (6) : There are 8 gaps among 7 boys. The three girls can be arranged in these 8 places in ${}^8P_3$ ways. Also the 7 boys can be arranged in 7!.

$\therefore$ The required number of permutations

$= 7! \times {}^8P_3 = 42 \cdot 8!$

$\therefore$ $K = 42$ and sum of the digits of K is $4 + 2 = 6$.

30. (7) : The alphabetical order of the word SACHIN is A C H I N S

If the first letter is from A, C, H, I, N, there are $5 \times 5! = 600$ words.

Then the next word is SACHIN

$\therefore$ The required serial number $K = 601$.

The sum of the digits of K is $6 + 0 + 1 = 7$.

31. (5) : The number of times A and B are together on guard is $\binom{n-2}{10}$.

The number of times C, D, E are together on guard is $\binom{n-3}{9}$.

$\Rightarrow \binom{n-2}{10} = 3\binom{n-3}{9}$

$\Rightarrow n - 2 = 30$

$\Rightarrow n = 32$

$\therefore$ The sum of the digits of n is $3 + 2 = 5$.

32. (7) : Here $a = {}^8C_3$ and $b = {}^7C_2$

$\therefore$ $a + b = 56 + 21 = 77$

The digit in unit place is 7.

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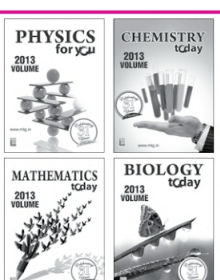

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PRACTICE PAPER 2014

JEE Main & Advanced

SECTION-I

Single Correct Answer Type

This section contains 8 single correct choice type questions. Each question has 4 choices: (a), (b), (c) and (d) for its answer, out of which **ONLY ONE** is correct.

1. If the curves $x^2 - y^2 = 4$ and $xy = \sqrt{5}$ intersect at points A and B, then the possible number of point(s) C on the curve $x^2 - y^2 = 4$ such that triangle ABC is equilateral is

(a) 0 (b) 1 (c) 2 (d) 4

2. If A and B are two events such that $P(A \cap B) = 0.3$ and $P(A' \cap B') = 0.6$, then the value of $P(A \cap B'$ or $A' \cap B)$ is equal to

(a) 0.9 (b) 0.7 (c) 0.3 (d) 0.1

3. The value of $\lim_{x\to\infty}\left(\frac{\int_0^{2x} x e^{x^2}\,dx}{e^{4x^2}}\right)$ equals

(a) $\frac{1}{2}$ (b) 1 (c) 2 (d) ∞

4. The number of values of k for which the equation $x^3 - 3x + k = 0$ has two distinct roots lying in the interval (0, 1) are

(a) three (b) two
(c) infinitely many
(d) no value of k satisfies the requirement

5. The domain of the function $f(x) = \cos^{-1}(\cos^{-1} x)$ is

(a) $[-1, 1]$ (b) $[0, \pi]$
(c) $[0, \cos 1]$ (d) $[\cos 1, 1]$

6. If $\frac{dy}{dx} = f(x) + \int_0^1 f(x)\,dx$, then the equation of the curve $y = f(x)$ passing through (0, 1) is

(a) $f(x) = \frac{2e^x - e + 1}{3 - e}$ (b) $f(x) = \frac{3e^x - 2e + 1}{2(2 - e)}$

(c) $f(x) = \frac{e^x - 2e + 1}{e + 1}$ (d) none of these

7. If the complex number z satisfies the equation $z\bar{z} = z^2 + \bar{z}^2$ then the maximum value of $\arg z - \arg \bar{z}$ is

(a) $\frac{5\pi}{3}$ (b) $\frac{4\pi}{3}$ (c) $\frac{2\pi}{3}$ (d) $\frac{\pi}{3}$

8. If a, b, c are in A.P. and $f(x) = \begin{vmatrix} x+a & x^2+1 & 1 \\ x+b & 2x^2-1 & 1 \\ x+c & 3x^2-2 & 1 \end{vmatrix}$, then $f'(1)$ is

(a) -1 (b) 0
(c) 1 (d) none of these

SECTION-II

Multiple Correct Answers Type

This section contains 4 multiple correct choice type questions. Each question has 4 choices: (a), (b), (c) and (d) for its answer, out of which **ONE** OR **MORE** is/ are correct.

9. Let P is any point on the curve $S = 0$ such that tangent from P to $x^2 + y^2 - 2x - 4y - 4 = 0$ makes 60° with each other and from point Q perpendicular tangents are drawn to S, then

(a) locus of P is circle of radius 5
(b) locus of P is a circle of radius 6
(c) locus of Q is a circle of radius $5\sqrt{2}$
(d) locus of Q is a circle of radius $6\sqrt{2}$

10. A function $f : R \to R^+$ satisfies $f(x + y) = f(x) \cdot f(y)$, $\forall x, y \in R$, $f(0) = 1$, $f'(0) = 2$ then

(a) $\int_0^{\ln 3} [f(x)e^{-x}]\,dx = \ln 4.5$ (where [.] denotes greatest integer function)

(b) $\lim_{x\to 0}[f(x)]$ does not exist (where [.] denotes greatest integer function)

(c) $f^{-1}(x) = \ln\sqrt{x}$, $\forall x > 0$

(d) $f(x) < e^{x^2 - 4x}$ has infinite solution in (0, 6)

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11. Consider the two lines $\vec{r} = \hat{i} + \hat{j} + \lambda_1(\hat{i} - \hat{k})$ and $\vec{s} = \hat{j} + \hat{k} + \lambda_2(\hat{i} + \hat{j})$, if PQ is the line of shortest distance, then

(a) their shortest distance is $\sqrt{26}$

(b) their point of intersection is $\hat{j} + \hat{k}$

(c) their shortest distance is zero

(d) the length of projection of $\overrightarrow{OP}$ on $\hat{i} + \hat{j} + \hat{k}$ is $\frac{2}{\sqrt{3}}$

12. Consider $n = 21^{52}$, then

(a) number of even divisors of n is 704

(b) number of divisors of n is 2809

(c) last two digits of n are 41

(d) number of divisors of n which is multiple of 9 is 2705

SECTION-III

Comprehension Type

This section contains 2 groups of questions. Each group has 3 multiple choice questions based on a paragraph. Each question has 4 choices: (a), (b), (c) and (d) for its answer, out of which **ONLY ONE** is correct.

Paragraph for Question Nos. 13 to 15

Consider an ellipse $S_1 \equiv \frac{x^2}{a^2} + \frac{y^2}{a^2(1-e^2)} - 1 = 0$.

Let $S_2 = 0$ be a parabola on the right of y-axis confocal with $S_1 = 0$ having vertex at the centre of $S_1 = 0$.

Let P be the point of intersection of the parabola and the directrix of ellipse in the 1st quadrant and $L = 0$ is the directrix of the parabola.

13. The ordinate of the point from which the pair of tangents drawn to both the parabola and the ellipse, are separately at right angles is

(a) $a\sqrt{1-e^2}$ (b) $\sqrt{2}a\sqrt{1-e^2}$

(c) $2a\sqrt{1-e^2}$ (d) $a\sqrt{2-e^2}$

14. Pair of tangents are drawn from any point on $L = 0$ to $S_1 = 0$ and $S_2 = 0$. Then the locus of point of intersection of their chord of contact is

(a) $x = ae$ (b) $x = \frac{a}{e}$

(c) $x = \frac{a\sqrt{1-e^2}}{e}$ (d) $x = ae\sqrt{1-e^2}$

15. If the tangent at point Q on $S_1 = 0$ and the line joining the points P to the focus of $S_2 = 0$ intersect at the auxiliary circle of $S_1 = 0$, then eccentric angle of the point Q is (Q lies in 1st quadrant)

(a) $\tan^{-1}\left(\frac{e}{\sqrt{1+e^2}}\right)$ (b) $\tan^{-1}\left(\frac{2e}{\sqrt{1-e^2}}\right)$

(c) $\tan^{-1}\left(\frac{\sqrt{2}e}{\sqrt{1+e^2}}\right)$ (d) $\tan^{-1}\left(\frac{e}{\sqrt{1-e^2}}\right)$

Paragraph for Question Nos. 16 to 18

Let α, β, γ be three real numbers such that $\alpha^2 + \beta^2 + \gamma^2 - \gamma = 0$ and $z = \frac{\alpha + \beta i}{1 - \gamma}$.

16. $|z|^2$ equals

(a) γ (b) $1 - \gamma$

(c) $\frac{\gamma}{1-\gamma}$ (d) $\frac{1-\gamma}{\gamma}$

17. α equals

(a) $\frac{z + \bar{z}}{2(1+|z|^2)}$ (b) $\frac{(\bar{z} + z)i}{2(1+|z|^2)}$

(c) $\frac{(z - \bar{z})i}{2(1+|z|^2)}$ (d) none of these

18. β equals

(a) $\frac{z - \bar{z}}{2(1+|z|^2)}$ (b) $\frac{(\bar{z} - z)i}{2(1+|z|^2)}$

(c) $\frac{z}{2(1+|z|^2)}$ (d) $\frac{2z}{1+|z|^2}$

SECTION-IV

Matrix Match Type

This section contains 2 questions. Each question contains statements given in two columns, which have to be matched. The statements in Column I are labelled A, B, C and D, while the statements in Column II are labelled 1, 2, 3, 4 and 5. Any given statement in Column I can have correct matching with ONE OR MORE statement(s) in Column II. The appropriate bubbles corresponding to the answers to these questions have to be darkened as illustrated in the following example:

	1	2	3	4	5
A	(1)	2	3	(4)	5
B	1	(2)	(3)	4	(5)
C	(1)	(2)	3	4	5
D	1	2	3	(4)	5

If the correct matches are A – 1, 4 and 5; B – 2, 3 and 5; C – 1, 2 and 5; D – 4; then the correct darkening of bubbles will look like the following.

19. A function is defined as $f(\alpha) = (-1)^{\alpha_1} + (-1)^{\alpha_2} + (-1)^{\alpha_3} + \ldots (-1)^{\alpha_k}$ where $\alpha \in N$, and $\alpha_1, \alpha_2, \alpha_3, \ldots, \alpha_k$ are all divisors of α including 1 and α itself.

Column-I		Column-II	
(A)	If $f(\alpha) = 4$ and $\alpha < 60$, then number of possible values of α are	1.	4
(B)	If $f(\alpha) = 0$, then α can be	2.	14
(C)	If $f(\alpha) < 0$, than α can't be	3.	25
(D)	If $f(\alpha) = -(2k+1)$, where $k \in$ natural number, then α can be	4.	6
		5.	10

20.

Column-I		Column-II	
(A)	If maximum and minimum values of $5\cos\theta+3\cos\left(\theta+\frac{\pi}{3}\right)+4$ for all real values of θ are m and n respectively, then	1.	$m+n=12$
(B)	If minimum and maximum values of $4+\sin\left(\frac{\pi}{4}+\theta\right)+2\cos\left(\frac{\pi}{4}-\theta\right)$ for all real values of θ are n and m respectively, then	2.	$m-n=6$
(C)	If number of solution of $\ln\lvert\sin x\rvert = x^2 - 16x$, when $x \in [0, \pi]$ is n and when $[0, 5\pi]$ is m, then	3.	$m+n=8$
(D)	If number of solutions of $1 + [\sin x] = \cos x$, (where [.] denotes the greatest integer function) when $x \in (0, 10\pi]$ is m	4.	$m-n=8$
		5.	$m-n=14$

SOLUTIONS

1. (a): A and B are $(\sqrt{5}, 1)$ and $(-\sqrt{5}, -1)$.
Let C be $(2\sec\theta, 2\tan\theta)$
$O(0, 0)$ is the mid point of AB

Slope of OC = $\sin\theta$ and slope of $AB = \frac{1}{\sqrt{5}}$
Since $OC \perp AB$
So, $\sin\theta = -\sqrt{5}$ which is impossible.

2. (d): $P(A' \cap B') = 0.6$
$\Rightarrow P(A \cup B') = 0.6$
$\Rightarrow P(A \cup B) = 0.4$
$\therefore P(A \cap B' \text{ or } A' \cap B)$
$= P(A \cap B') + P(A' \cap B)$
$= P(A \cup B) - P(A \cap B) = 0.4 - 0.3 = 0.1$

3. (a): $\lim\limits_{x\to\infty} \dfrac{\int_0^{2x} x e^{x^2} dx}{e^{4x^2}}$

$= \lim\limits_{x\to\infty} \dfrac{2x.e^{4x^2}.2}{e^{4x^2}.8x}$ (Using L' Hospital rule)

$= \dfrac{1}{2}$

4. (d): Let there be a value of k for which $x^3 - 3x + k = 0$ has two distinct roots between 0 and 1.

Let a, b be two distinct roots of $x^3 - 3x + k = 0$ lying between 0 and 1 such that $a < b$. Let $f(a) = f(b) = 0$. Since between any two roots of a polynomial $f(x)$ there exists at least one roots of its derivative $f'(x)$. Therefore $f'(x) = 3x^3 - 3$ has at least one root between a and b. But $f'(x) = 0$ has two roots equal to ± 1 which do not lie between a and b. Hence $f(x) = 0$ has no real roots lying between 0 and 1 for any value of k.

5. (d): $0 \le \cos^{-1} x \le 1$
$x \in [\cos 1, 1]$

6. (a): $f''(x) = f'(x) \Rightarrow \dfrac{f''(x)}{f'(x)} = 1$
On integrating,
$f'(x) = Ce^x$
Which gives $f(x) = Ce^x + D$
But $f(0) = 1 \Rightarrow C + D = 1$
$\therefore f(x) = Ce^x + 1 - C$
So, $f'(x) = Ce^x$
Putting it in $f'(x) = f(x) + \int_0^1 f(x)dx$

$\Rightarrow Ce^x = Ce^x + 1 - C + \int_0^1 (Ce^x + 1 - C)dx$

$\Rightarrow C = \dfrac{2}{3-e}$

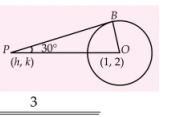
So, $f(x) = \dfrac{2e^x - e + 1}{3 - e}$

7. (a): Let $z = x + iy \Rightarrow y = \pm\dfrac{1}{\sqrt{3}}x$

So, maximum value of
$\arg z - \arg \bar{z} = \dfrac{5\pi}{6} - \left(\dfrac{-5\pi}{6}\right) = \dfrac{5\pi}{3}$.

8. (b)

9. (b, d):



$\dfrac{1}{\sqrt{3}} = \dfrac{3}{\sqrt{h^2 + k^2 - 2h - 4k - 4}}$

$\Rightarrow x^2 + y^2 - 2x - 4y - 31 = 0$
So locus of P is a circle of radius 6
Locus of Q is $(x+1)^2 + (y+2)^2 = \left(6\sqrt{2}\right)^2$
So locus of Q is a circle of radius $\left(6\sqrt{2}\right)$

10. (a, b, c): $f(x) = e^{2x}$

(a) $\int_0^{\ln 3} [e^x]\,dx = \ln 2 + 2(\ln 3 - \ln 2) = \ln 9 - \ln 2 = \ln 4.5$

(b) $\lim_{x\to 0^+}[e^{2x}]=1;\ \lim_{x\to 0^-}[e^{2x}]=0$

$\Rightarrow \lim_{x\to 0}[e^{2x}]$ does not exist

(c) $f^{-1}(x)=\ln\sqrt{x},\ x>0$

(d) $e^{2x}<e^{x^2-4x};\ 2x<x^2-4x$

$\Rightarrow\ x^2-6x>0\ \Rightarrow\ x<0$ or $x>6$

11. (b, c, d): Let $\vec{P}=\hat{i}(\lambda_1+1)+\hat{j}-\lambda_1\hat{k}$;

$\vec{Q}=\lambda_2\hat{i}+\hat{j}(1+\lambda_2)+\hat{k}$

$\overrightarrow{PQ}=\hat{i}(\lambda_2-\lambda_1-1)+\hat{j}\lambda_2+\hat{k}(1+\lambda_1)$

$\overrightarrow{PQ}.(\hat{i}-\hat{k})=0$ also $\overrightarrow{PQ}.(\hat{i}+\hat{j})=0$

$\lambda_2-\lambda_1-1-1-\lambda_1=0;\ \lambda_2-\lambda_1-1+\lambda_2=0$

$\lambda_2-2\lambda_1=2$ …(i)

$2\lambda_2-\lambda_1=1$ …(ii)

From (i) and (ii) $\Rightarrow\ \lambda_2=0,\ \lambda_1=-1$

$\overrightarrow{PQ}=\vec{0}$

$\overrightarrow{OQ}=\overrightarrow{OP}=\hat{j}+\hat{k}$

So projection of $\overrightarrow{OP}$ on $\hat{i}+\hat{j}+\hat{k}$

$=(\hat{i}+\hat{j}).\dfrac{(\hat{i}+\hat{j}+\hat{k})}{\sqrt{3}}=\dfrac{2}{\sqrt{3}}$

12. (b, c):

(b) $n=3^{52}\cdot 7^{52}$

So number of divisors $53\times 53=2809$

(c) $21^{52}=(20+1)^{52}$

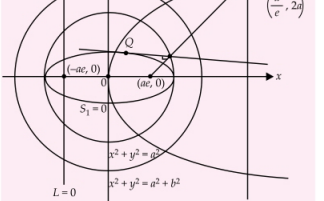
$={}^{52}C_0\,20^{52}+{}^{52}C_1\,20^{51}+\ldots+{}^{52}C_{50}\,20^2+{}^{52}C_{21}\,20+1$

$=100k+52\times 20+1$

$=100k+1041=100k+41$

(d) number of divisor which is multiple of 9.

$=53\times 51=2703$



13. (b): The point from which the pair of tangents drawn to both the curves are at right angles will be found by solving $x=-ae$ and $x^2+y^2=a^2+b^2$ *i.e.*, $(-ae, \sqrt{2}a\sqrt{1-e^2})$

14. (b): Any point of $L=0$ is $(-ae, \alpha)$.

Chord of contact of $S_1=0$ is $\dfrac{-ex}{a}+\dfrac{y\alpha}{a^2(1-e^2)}=1$

and Chord of contact of $S_2=0$ is $y\alpha=2ae\,(x-ae)$

Solving and eliminating α, we have $x=\dfrac{a}{e}$

15. (b): Let Q be $(a\cos\theta, a\sqrt{1-e^2}\sin\theta)$

Slope of tangent at point Q to the ellipse is $(-\sqrt{1-e^2}\cot\theta)$

But tangent at Q will be perpendicular to the line joining P to the focus of parabola.

So, $\left(\dfrac{\dfrac{2a-0}{a}}{\dfrac{a}{e}-ae}\right)\times[-\sqrt{1-e^2}\cdot\cot\theta]=-1$

$\Rightarrow\ \dfrac{2e}{1-e^2}\times\sqrt{1-e^2}\cot\theta=1$

$\Rightarrow\ \tan\theta=\dfrac{2e}{\sqrt{1-e^2}}$

$\therefore\ \theta=\tan^{-1}\left(\dfrac{2e}{\sqrt{1-e^2}}\right)$

16. (c): $|z|^2=\dfrac{\alpha^2+\beta^2}{(1-\gamma)^2}=\dfrac{\gamma-\gamma^2}{(1-\gamma)^2}=\dfrac{\gamma}{1-\gamma}$

17. (a): $|z|^2=\dfrac{\gamma}{1-\gamma}=-1+\dfrac{1}{1-\gamma}$

$\Rightarrow\ 1+|z|^2=\dfrac{1}{1-\gamma}\Rightarrow\gamma=\dfrac{|z|^2}{1+|z|^2}$

Now, $\dfrac{2\alpha}{1-\gamma}=z+\bar{z}$

$\Rightarrow\ 2\alpha=(z+\bar{z})(1-\gamma)=\dfrac{z+\bar{z}}{1+|z|^2}$

$\Rightarrow\ \alpha=\dfrac{z+\bar{z}}{2(1+|z|^2)}$

18. (b): $\dfrac{2i\beta}{1-\gamma}=z-\bar{z}$

$\Rightarrow\ 2i\beta=(z-\bar{z})(1-\gamma)=\dfrac{z-\bar{z}}{1+|z|^2}$

$\Rightarrow \quad \beta = \dfrac{(\bar{z} - z)i}{2(1 + |z|^2)}$

19. A – 1; B - 2, 4, 5; C - 1, 2, 4, 5; D - 3

If $N = 2^{K_0} p_1^{K_1} p_2^{K_2} p_3^{K_3} \ldots\ldots p_n^{K_n}$

Total number of divisors are =

$$(K_0 + 1)(K_1 + 1)(K_2 + 1)\ldots(K_n + 1)$$

Total number of even divisors =

$$K_0(K_1 + 1)(K_2 + 1)\ldots(K_n + 1)$$

Total number of odd divisors =

$$(K_1 + 1)(K_2 + 1)\ldots(K_n + 1)$$

(A) $f(\alpha) = 4 \Rightarrow (K_0 - 1)(K_1 + 1)(K_2 + 1)\ldots(K_n + 1) = 4$

Case-1: $K_0 = 5 \Rightarrow N = 2^5$

Case-2: $K_0 = 3$, $N = 2^3 3$, $2^3 5$, $2^3 7$

Case-3: $K_0 = 2 \Rightarrow$ not possible

(B) $f(\alpha) = 0 \Rightarrow K_0 = 1$

(C) for $f(\alpha) < 0 \Rightarrow K_0 = 0$

(D) $f(\alpha) = -(2K + 1)$, then $K_0 = 0$ and $K_1, K_2, \ldots$ must be even

20. (A) – 3, 5 ; (B) – 2, 3 ; (C) – 1, 4 ; (D) – 1, 4

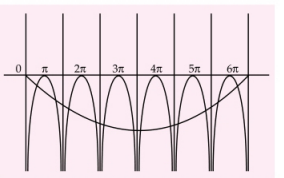
(A) $5\cos\theta + 3\cos\left(\theta + \dfrac{\pi}{3}\right) + 4 = 7\cos(\theta + \alpha) + 4$

$\Rightarrow \quad m = 11, n = -3$

(B) $4 + \sin\left(\dfrac{\pi}{4} + \theta\right) + 2\cos\left(\dfrac{\pi}{4} - \theta\right)$

$= 4 + 3\cos\left(\theta - \dfrac{\pi}{4}\right) \quad \Rightarrow m = 7, \; n = 1$

(C)



$\Rightarrow \quad m = 10, n = 2$

(D) $x = \dfrac{(4k-1)\pi}{2}$ or $2k\pi$

$\Rightarrow \quad m = 10, n = 2$

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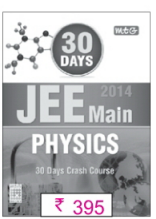

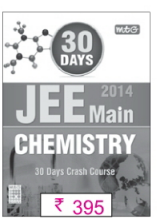

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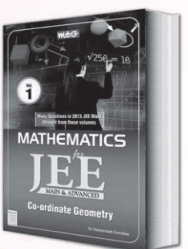

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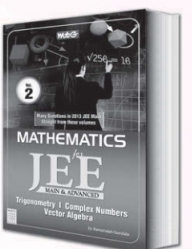

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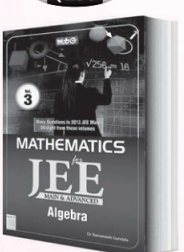

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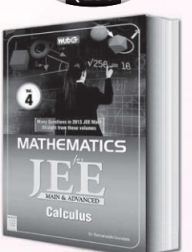

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JEE (Main & Advanced) and other PETs

PAPER-I

SECTION-1

SINGLE CORRECT CHOICE TYPE

This section contains 6 multiple choice questions. Each question has four choices (a), (b), (c) and (d) for its answer, out of which ONLY ONE is correct. [Correct ans. 3 marks & wrong ans. –1]

1. In the sequence 1, 2, 2, 4, 4, 4, 4, 8, 8, 8, 8, 8, 8, 8, 8, …., where n consecutive terms have the value n, the 1025^{th} term is
(a) 2^9 (b) 2^{10}
(c) 2^{11} (d) 2^8

2. If $a_n > 1$ for all $n \in N$ then $\log_{a_2} a_1 + \log_{a_3} a_2 + ... + \log_{a_n} a_{n-1} + \log_{a_1} a_n$ has the minimum value
(a) 1 (b) 2
(c) 0 (d) none of these

3. If $|z| = \max\{|z-1|, |z+1|\}$ then
(a) $|z+\bar{z}| = \frac{1}{2}$ (b) $z+\bar{z} = 1$
(c) $|z+\bar{z}| = 1$ (d) none of these

4. If $\theta \in [0, 5\pi]$ and $r \in R$ such that $2\sin\theta = r^4 - 2r^2 + 3$ then the maximum number of values of the pair (r, θ) is
(a) 8 (b) 10
(c) 6 (d) none of these

5. The number of real solutions of
$$\tan^{-1}\sqrt{x(x+1)} + \sin^{-1}\sqrt{x^2+x+1} = \frac{\pi}{2} \text{ is}$$
(a) zero (b) one
(c) two (d) infinite

6. In a ΔABC, $\cos B \cdot \cos C + \sin B \cdot \sin C \cdot \sin^2 A = 1$. Then the triangle is
(a) right-angled isosceles
(b) isosceles whose equal angles are greater than $\pi/4$
(c) equilateral
(d) none of these

SECTION-II

MULTIPLE CORRECT CHOICE TYPE

This section contains 4 multiple choice questions. Each question has four choices (a), (b), (c) and (d) out of which ONE OR MORE may be correct. [Correct ans. 4 marks & wrong ans., no negative mark]

7. From a point P, the chord of contact to the ellipse
$$\frac{x^2}{a} + \frac{y^2}{b} = (a+b) \quad \text{... (1)}$$
touches the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ ... (2)
then the locus of the point P is
(a) director circle of (1)
(b) auxiliary circle of (2)
(c) $x^2 + y^2 = (a+b)^2$
(d) $x^2 + y^2 = a^2 + b^2$

8. On the interval $I = [-2, 2]$ the function
$$f(x) = \begin{cases} (x+1)e^{-\left(\frac{1}{|x|}+\frac{1}{x}\right)}, & x \neq 0 \\ 0, & x = 0 \end{cases}$$
(a) is continuous for all $x \in I - \{0\}$
(b) is continuous for all $x \in I$
(c) assumes all intermediate values for $f(-2)$ to $f(2)$
(d) has a maximum values equal to $3/e$

9. The point $P(\alpha, \alpha+1)$ will lie inside the ΔABC whose vertices are $A(0, 3)$, $B(-2, 0)$ and $C(6, 1)$ if
(a) $\alpha = -1$ (b) $\alpha = -\frac{1}{2}$
(c) $\alpha = \frac{1}{2}$ (d) $-\frac{6}{7} < \alpha < \frac{3}{2}$

10. If $x^2 + mx + 1 = 0$ and $(b-c)x^2 + (c-a)x + (a-b) = 0$ have both roots common then
(a) $m = -2$ (b) $m = -1$
(c) a, b, c are in A.P. (d) a, b, c are in H.P.

By : **Vidyalankar Institute**, Pearl Centre, Senapati Bapat Marg, Dadar (W), Mumbai - 28. Tel.: (022) 24306367

SECTION-III

REASONING TYPE

This section contains 4 reasoning type questions. Each question has 4 choices (a), (b), (c) and (d), out of which ONLY ONE is correct. [Correct ans. 3 marks & wrong ans. –1]

(a) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1

(b) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1

(c) Statement-1 is True, Statement-2 is False.

(d) Statement-1 is False, Statement-2 is True.

11. Given two straight lines whose equations are

$$\frac{x-3}{1}=\frac{y-5}{-2}=\frac{z-7}{1} \text{ and } \frac{x+1}{7}=\frac{y+1}{-6}=\frac{z+1}{1}$$

Statement-1 : The line of shortest distance between the given lines is perpendicular to the plane $x + 3y + 5z = 0$.

Statement-2 : The direction ratios of the normal to the plane $ax + by + cz + d = 0$ are $\frac{a}{d}, \frac{b}{d}, \frac{c}{d}$.

12. Statement-1 : The probability of getting a tail most of the time in 10 tosses of an unbiased coin is $\frac{1}{2}\left\{1-\frac{10!}{2^{10}\,5!\,5!}\right\}$.

Statement-2 : $^{2n}C_0 + {}^{2n}C_1 + {}^{2n}C_2 + \ldots + {}^{2n}C_n = 2^{2n-1}, n \in N$.

13. Let a, b, c be real such that $ax^2 + bx + c = 0$ and $x^2 + x + 1 = 0$ have a common root.

Statement-1 : $a = b = c$.

Statement-2 : Two quadratic equations with real coefficients cannot have only one imaginary root common.

14. Let $F(x) = \int_1^{x^2} \cos\sqrt{t}\, dt$

Statement-1 : $F'(x) = \cos x$.

Statement-2 : If $f(x) = \int_a^x \phi(t)dt$ then $f'(x) = \phi(x)$.

SECTION-IV

PARAGRAPH TYPE

This section contains 3 paragraph. Based upon each of the paragraphs 3 multiple choice questions have to be answered. Each of these questions has four choices (a), (b), (c) and (d) out of which ONLY ONE is correct. [Correct ans. 4 marks & wrong ans. –1]

Paragraph for Q. Nos. 15 to 17

Perpendicular are drawn from the focus S of the parabola $y = ax^2 + bx + c$ upon the tangents to the parabola at the points $A(-1, 0)$ and $B(1, 2)$ meeting them at the point $C\left(-\frac{1}{4}, \frac{9}{4}\right)$ and $D\left(\frac{3}{4}, \frac{9}{4}\right)$ respectively.

15. The co-ordinates of the focus are

(a) $\left(\frac{1}{2}, \frac{9}{4}\right)$ (b) $\left(\frac{1}{2}, 2\right)$

(c) $\left(0, \frac{9}{4}\right)$ (d) $(-1, 2)$

16. The normals at A and B intersect at a point P. The foot of the third normal through the point P is

(a) $\left(\frac{1}{2}, \frac{9}{4}\right)$ (b) $\left(-\frac{1}{2}, \frac{5}{4}\right)$

(c) $(0, 2)$ (d) $\left(\frac{3}{2}, \frac{5}{4}\right)$

17. Area of the region bounded by the parabola and the x-axis is

(a) $\frac{5}{4}$ (b) 5 (c) $\frac{5}{2}$ (d) $\frac{9}{2}$

Paragraph for Q. Nos. 18 to 20

Let an ellipse having major axis and minor axis parallel to x-axis and y-axis respectively. Its two foci S and S' are (2, 1), (4, 1) and a line $x + y = 9$ is a tangent to this ellipse at point P.

18. Eccentricity of the ellipse is

(a) $\frac{1}{\sqrt{12}}$ (b) $\frac{1}{\sqrt{13}}$

(c) $\frac{1}{2}$ (d) none of these

19. Length of major axis is

(a) $\sqrt{13}$ (b) $2\sqrt{11}$ (c) $\sqrt{52}$ (d) $2\sqrt{12}$

20. The latus rectum of ellipse

(a) $\frac{12}{13}$ (b) $\frac{12}{\sqrt{13}}$ (c) $\frac{24}{\sqrt{13}}$ (d) $\frac{25}{\sqrt{13}}$

Paragraph for Q. Nos. 21 to 23

The functions $\sin^{-1}x$, $\cos^{-1}x$, $\tan^{-1}x$, $\cot^{-1}x$, $\operatorname{cosec}^{-1}x$ and $\sec^{-1}x$ are called inverse circular or inverse trigonometric functions which are defined as follows:

$\sin^{-1}x$	$-1 \le x \le 1$	$\frac{\pi}{2} \le \sin^{-1}x \le \frac{3\pi}{2}$
$\cos^{-1}x$	$-1 \le x \le 1$	$-\pi \le \cos^{-1}x \le 0$
$\tan^{-1}x$	$x \in R$	$-\frac{\pi}{2} < \tan^{-1}x < \frac{\pi}{2}$

$\text{cosec}^{-1}x$	$\lvert x\rvert \geq 1$	$\frac{\pi}{2} \leq \text{cosec}^{-1}x \leq \frac{3\pi}{2} \neq \pi$
$\sec^{-1}x$	$\lvert x\rvert \geq 1$	$-\pi \leq \sec^{-1}x \leq 0 \neq -\frac{\pi}{2}$
$\cot^{-1}x$	$x \in R$	$0 < \cot^{-1}x < \pi$

21. For $x \in [0, 1]$, $\sin^{-1}x$ is equal to

(a) $\cos^{-1}\sqrt{1-x^2} + \pi$ (b) $\cos^{-1}\sqrt{1-x^2} + \frac{\pi}{2}$

(c) $-\cos^{-1}\sqrt{1-x^2}$ (d) none of these

22. Number of solutions of $\tan^{-1}|x| - \cos^{-1}x = 0$ is/are

(a) 2 (b) 1

(c) 0 (d) none of these

23. $\lim_{x\to 0} \tan\left(\frac{\sin^{-1}x^4 + \sin^{-1}x^9}{4}\right)$

(a) does not exist as L.H.L and R.H.L. both are finite and unequal

(b) exist as L.H.L. = R.H.L.

(c) R.H.L and L.H.L. are unequal

(d) none of these

PAPER-II

SECTION-I

SINGLE CORRECT CHOICE TYPE

This section contains 9 multiple choice questions. Each question has four choices (a), (b), (c) and (d) for its answer, out of which ONLY ONE is correct. [Correct ans. 3 marks & wrong ans. –1]

1. Two circles have the equations $x^2 + y^2 - 4x - 6y - 8 = 0$ and $x^2 + y^2 - 2x - 3 = 0$. Then

(a) they cut each other

(b) they touch each other

(c) one circle lies inside the other

(d) one circle lies wholly outside the other

2. Let the r^{th} term, t_r, of a series is given by $t_r = \frac{r}{1 + r^2 + r^4}$. Then $\lim_{n\to\infty} \sum_{r=1}^{n} t_r$ is

(a) 1/4 (b) 1

(c) 1/2 (d) none of these

3. In a ΔABC, $\angle B = 90°$ and $b + a = 4$. The area of the triangle is the maximum when $\angle C$ is

(a) $\pi/4$ (b) $\pi/6$

(c) $\pi/3$ (d) none of these

4. There are 4 white and 3 black balls in a box. In another box there are 3 white and 4 black balls. An unbiased dice is rolled. If it shows a number less than or equal to 3 then a ball is drawn from the first box but if it shows a number more than 3 then a ball is drawn from the second box. If the ball drawn is black then the probability that the ball was drawn from the first box is

(a) 1/2 (b) 6/7 (c) 4/7 (d) 3/7

5. The number of non-integral solutions of $||4x - x^2| - 1| = 3$ is

(a) four (b) two

(c) three (d) none of these

6. If the normal at any point P on an ellipse meets major and minor axis at G and G' and OF be the perpendicular drawn from centre O to this normal then PF, PG must be equal to

(a) b^2 (b) a^2

(c) ab (d) none of these

7. The normal at an end of latus rectum of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ passes through an end of the minor axis if

(a) $e^4 + e^2 = 1$ (b) $e^3 + e^2 = 1$

(c) $e^2 + e = 1$ (d) $e^3 + e = 1$

8. $\lim_{n\to\infty} \left(\frac{3n+8}{3n+5}\right)^{5n+9}$ is equal to

(a) $3e^5$ (b) e^5

(c) e^3 (d) none of these

9. Let $\overline{AB} = 3\hat{i} + \hat{j} - \hat{k}$ and $\overline{AC} = \hat{i} - \hat{j} + 3\hat{k}$. If the point P on the line segment BC is equidistant from AB and AC then $\overline{AP}$ is

(a) $2\hat{i} - \hat{k}$ (b) $\hat{i} - 2\hat{k}$

(c) $2\hat{i} + \hat{k}$ (d) none of these

SECTION-II

REASONING TYPE

This section contains 4 reasoning type questions. Each question has 4 choices (a), (b), (c) and (d), out of which ONLY ONE is correct. [Correct ans. 3 marks & wrong ans. –1]

(a) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1

(b) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1

(c) Statement-1 is True, Statement-2 is False.

(d) Statement-1 is False, Statement-2 is True.

10. Let $(1+x)^{36} = a_0 + a_1x + a_2x^2 + ... + a_{36}x^{36}$.

Statement-1 : $a_0 + a_3 + a_6 + ... + a_{36} = \frac{2}{3}(2^{35}+1)$

Statement-2 : $a_0 + a_1 + a_2 + ... + a_{36} = 2^{36}$ and $a_0 + a_2 + a_4 + ... + a_{36} = 2^{35}$

11. z is a unimolecular complex number.

Statement-1 : $\arg(z^2 + \bar{z}) = \arg z$.

Statement-2 : $\bar{z} = \cos(\arg z) - i\sin(\arg z)$

12. a, b, c are three unequal positive numbers.

Statement-1 : The product of their sum and the sum of their reciprocals exceeds 9.

Statement-2 : AM of n positive numbers exceeds their HM.

13. **Statement-1 :** $1 \cdot 3 \cdot 5 \cdot \ldots (2n-1) > n^n, n \in N$.

Statement-2 : The sum of the first n odd natural numbers is equal to n^2.

SECTION-III

PARAGRAPH TYPE

This section contains 2 paragraph. Based upon each of the paragraphs 3 multiple choice questions have to be answered. Each of these questions has four choices (a), (b), (c) and (d) out of which ONLY ONE is correct. [Correct ans. 4 marks & wrong ans. –1]

Paragraph for Q. No. 14 to 16

If z_1, z_2 be the complex numbers representing two points A and B, then we define the complex slope of the line AB as $\mu = \frac{z_1 - z_2}{z_1 - z_2}$, it can be noted that $|\mu| = 1$ and μ remains same for any two points on the line AB, since if $z_3 \cdot z_4$ be complex numbers representing some other points on the same line, then

$$\mu' = \frac{z_3 - z_4}{z_3 - z_4} = \frac{\lambda(z_1 - z_2)}{\lambda(z_1 - z_2)} \quad (\because z_3 - z_4 = \lambda(z_1 - z_2), \lambda \text{ real})$$

$$= \frac{z_1 - z_2}{z_1 - z_2} = \mu$$

14. The complex slope of the line $\bar{a}z + a\bar{z} + b = 0$, where a is complex and b is real is

(a) $\frac{a}{a}$ (b) $-\frac{a}{a}$ (c) $\frac{\bar{a}}{a}$ (d) $-\frac{\bar{a}}{a}$

15. If the complex slope of a line which is not parallel to y-axis is $\cos\phi + i\sin\phi$, then the line makes an angle θ with x-axis, θ must be

(a) 2ϕ (b) $90° - \phi$ (c) $\phi/2$ (d) ϕ

16. If μ and μ' be complex slopes of two perpendicular lines, then

(a) $\mu\mu' = 1$ (b) $\mu\mu' = -1$

(c) $\mu + \mu' = 0$ (d) none of these

Paragraph for Q. No. 17 to 19

Let $f : [2, \infty) \to [1, \infty)$ defined by $f(x) = 2^{x^4 - 4x^2}$ and $g : \left[\frac{\pi}{2}, \pi\right] \to A$ defined by $g(x) = \frac{\sin x + 4}{\sin x - 2}$ be two invertible functions, then

17. $f^{-1}(x)$ is equal to

(a) $-\sqrt{2 + \sqrt{4 + \log_2 x}}$ (b) $\sqrt{2 + \sqrt{4 + \log_2 x}}$

(c) $\sqrt{2 - \sqrt{4 + \log_2 x}}$ (d) none of these

18. The set A is equal to

(a) $[-5, -2]$ (b) $[2, 5]$

(c) $[-5, 2]$ (d) $[-3, -2]$

19. The domain of $f^{-1}g^{-1}(x)$ is

(a) $[-5, \sin 1]$ (b) $\left[-5, \frac{\sin 1}{2 - \sin 1}\right]$

(c) $\left[-5, \frac{(4 + \sin 1)}{2 - \sin 1}\right]$ (d) $\left[-\frac{(4 + \sin 1)}{2 - \sin 1}, -2\right]$

SECTION-IV

MATRIX-MATCH TYPE

This section contains 3 questions. Each question contains statements given in two columns, which have to be matched. The statements in Column-I are labelled A, B, C and D, while the statements in Column-II are labelled p, q, r and s. Any given statement in Column-I can have correct matching with ONE OR MORE statement(s) in Column-II. The appropriate bubbles corresponding to the answers to these questions have to be darkened as illustrated in the following example. If the correct matches are A-p and s ; B-q and r; C-p and q; and D-s ; then the correct darkening of bubbles will look like the following :

	p	q	r	s
A	(p)	(q)	(r)	(s)
B	(p)	(q)	(r)	(s)
C	(p)	(q)	(r)	(s)
D	(p)	(q)	(r)	(s)

[Correct ans. 2 mark for each row and maximum 8 marks, no negative marking]

20. Let $f : R \to R$ be a periodic function such that

Column I		Column II	
(A)	$f(T + x) = 1 + [1 - 3f(x) + 3(f(x))^2 - (f(x)^3]^{1/3}$, where T is fixed positive number, then period of $f(x)$ is AT, where A =	(p)	3
(B)	The area between the curve $y = 2x^4 - x^2$, the x-axis and the ordinates of two minima of the curve is $\frac{B}{120}$ where B is	(q)	2

(C)	$\int_0^4 \frac{f(x)}{f(x)+f(4-x)}\,dx =$	(r)	7
(D)	$f(x)=\begin{cases}\frac{1-\sin x}{(\pi-2x)^2}\,\frac{\log\sin x}{\log(1+\pi^2-4\pi x+4x^2)} \\ k, \qquad x=\frac{\pi}{2}\end{cases}$ is continuous at $x=\pi/2$, then $-1/k$ equals to	(s)	64

21. The parabola $y^2 = 4ax$ has a chord AB joining points $A(at_1^2, 2at_1)$ and $B(at_2^2, 2at_2)$. Then match the following.

Column I		Column II	
(A)	AB is normal chord	(p)	$t_2 = -t_1 - \frac{1}{2}$
(B)	AB is a focal chord	(q)	$t_2 = -\frac{4}{t_1}$
(C)	AB subtends 90° at point (0, 0)	(r)	$t_2 = -\frac{1}{t_1}$
(D)	AB is inclined at 45° to the axis of parabola	(s)	$t_2 = -t_1 - \frac{2}{t_1}$

22. Match the following:

Column I		Column II	
(A)	If $\lvert x^2 - x\rvert \geq x^2 + x$, then x	(p)	$[0, \infty)$
(B)	$\lvert x + y\rvert > x - y$, where $x > 0$, then $y =$	(q)	$(-\infty, 0)$
(C)	If $\log_2 x \geq \log_3 x^2$, then $x =$	(r)	$[-1, \infty)$
(D)	$[x] + 2 \geq \lvert x\rvert$ (where $[\cdot]$ denotes the greatest integer function)	(s)	$(0, 1]$

ANSWER KEY

PAPER-I

1.	(b)	2.	(d)	3.	(c)	4.	(c)	5.	(c)
6.	(a)	7.	(a, c)	8.	(a, c, d)				
9.	(b, c, d)			10.	(a, c)	11.	(d)	12.	(c)
13.	(a)	14.	(d)	15.	(b)	16.	(d)	17.	(d)
18.	(b)	19.	(c)	20.	(c)	21.	(a)	22.	(b)
23.	(c)								

PAPER-II

1.	(a)	2.	(c)	3.	(c)	4.	(d)	5.	(b)
6.	(a)	7.	(a)	8.	(b)	9.	(c)	10.	(b)
11.	(d)	12.	(a)	13.	(d)	14.	(b)	15.	(c)
16.	(c)	17.	(b)	18.	(a)	19.	(c)		

20. (A) → (q), (B) → (r), (C) → (q), (D) → (s)
21. (A) → (s), (B) → (r), (C) → (q), (D) → (p)
22. (A) → (q), (B) → (p), (C) → (s), (D) → (r)

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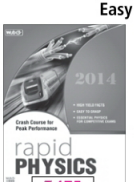

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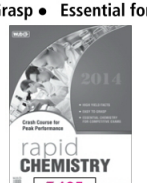

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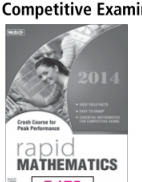

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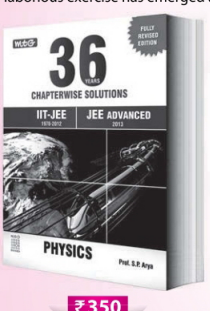

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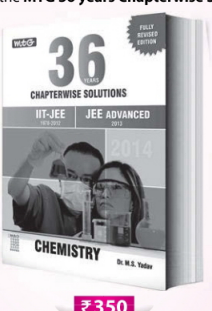

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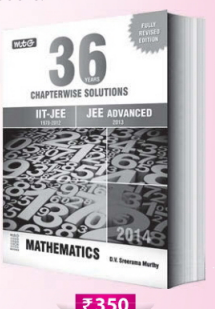

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concept BOOSTERS

Class XI

CIRCLES

* ALOK KUMAR, B.Tech, IIT Kanpur

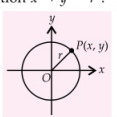
This column is aimed at Class XI students so that they can prepare for competitive exams such as JEE Main/Advanced, etc. and be also in command of what is being covered in their school as part of NCERT syllabus. The problems here are a happy blend of the straight and the twisted, the simple and the difficult and the easy and the challenging.

CONCEPTS AND FORMULAE

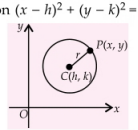
- A circle is a locus of a point whose distance from a fixed point (called centre) is always constant (called radius).

EQUATION OF A CIRCLE IN VARIOUS FORMS

(i) The circle with centre as origin and radius 'r' has the equation $x^2 + y^2 = r^2$.



(ii) The circle with centre (h, k) and radius 'r' has the equation $(x - h)^2 + (y - k)^2 = r^2$.



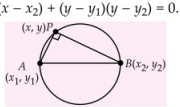
(iii) The general equation of a circle is
$x^2 + y^2 + 2gx + 2fy + c = 0$

with centre as $(-g, -f)$ and radius $= \sqrt{g^2 + f^2 - c}$

This can be obtained from the equation
$(x - h)^2 + (y - k)^2 = r^2$
$\Rightarrow x^2 + y^2 - 2hx - 2ky + h^2 + k^2 - r^2 = 0$
Take $-h = g$, $-k = f$ and $h^2 + k^2 - r^2 = c$
Also if
$g^2 + f^2 - c > 0 \Rightarrow$ real circle.
$g^2 + f^2 - c = 0 \Rightarrow$ point circle.
$g^2 + f^2 - c < 0 \Rightarrow$ imaginary circle, with real centre, that is $(-g, -f)$

Every second degree equation in x & y, in which coefficient of x^2 is equal to coefficient of y^2 & the coefficient of xy is zero, always represents a circle.

(iv) The equation of a circle with (x_1, y_1) & (x_2, y_2) as extremities of its diameter is:
$(x - x_1)(x - x_2) + (y - y_1)(y - y_2) = 0$.



This is obtained by the fact that angle in a semicircle is a right angle.
$\therefore$ (Slope of PA) (Slope of PB) $= -1$

$$\Rightarrow \frac{y - y_1}{x - x_1} \cdot \frac{y - y_2}{x - x_2} = -1$$

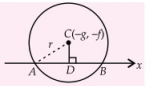
Note that this will be the circle of least radius passing through (x_1, y_1) & (x_2, y_2).

INTERCEPTS MADE BY A CIRCLE ON THE AXES

- The intercepts made by the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ on the co–ordinate axes are $2\sqrt{g^2 - c}$ (on x-axis) & $2\sqrt{f^2 - c}$ (on y-axis) respectively.

If $g^2 > c \Rightarrow$ circle cuts the x-axis at two distinct points.
$g^2 = c \Rightarrow$ circle touches the x–axis.
$g^2 < c \Rightarrow$ circle lies completely above or below the x–axis.

* Alok Kumar is a winner of INDIAN NATIONAL MATHEMATICS OLYMPIAD (INMO-91). He trains IIT and Olympiad aspirants.

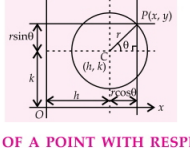


$$AB = 2AD = 2\sqrt{r^2 - CD^2} = 2\sqrt{r^2 - f^2}$$

$$= 2\sqrt{g^2 + f^2 - c - f^2} = 2\sqrt{g^2 - c}$$

PARAMETRIC EQUATIONS OF A CIRCLE

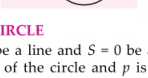
The parametric equations of $(x - h)^2 + (y - k)^2 = r^2$ are: $x = h + r \cos\theta$; $y = k + r \sin\theta$; $0 < \theta < 2\pi$ where (h, k) is the centre, r is the radius & θ is a parameter.



POSITION OF A POINT WITH RESPECT TO A CIRCLE

The point (x_1, y_1) is inside, on or outside the circle $S \equiv x^2 + y^2 + 2gx + 2fy + c = 0$ according as $S_1 \equiv x_1^2 + y_1^2 + 2gx_1 + 2fy_1 + c <$, = or > 0.

The greatest and the least distance of a point A from a circle with centre C & radius r is $AC + r$ & $AC - r$ respectively.



LINE AND A CIRCLE

Let $L = 0$ be a line and $S = 0$ be a circle. If r is the radius of the circle and p is the length of the perpendicular from the centre on the line, then:

(i) $p > r \Leftrightarrow$ the line does not meet the circle *i.e.* passes outside the circle.

(ii) $p = r \Leftrightarrow$ the line touches the circle. (It is tangent to the circle)

(iii) $p < r \Leftrightarrow$ the line is a secant of the circle.

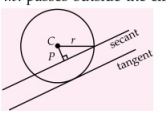
(iv) $p = 0 \Rightarrow$ the line is a diameter of the circle.

Also, if $y = mx + c$ is a line and $x^2 + y^2 = a^2$ is a circle then

(i) $c^2 > a^2(1 + m^2) \Rightarrow$ the line is a secant of the circle.

(ii) $c^2 = a^2(1 + m^2) \Rightarrow$ the line touches the circle. (It is tangent to the circle)

(iii) $c^2 < a^2(1 + m^2) \Rightarrow$ the line does not meet the circle *i.e.* passes outside the circle.



These conditions can also be obtained by solving $y = mx + c$ with $x^2 + y^2 = a^2$ and making the discriminant of the quadratic greater than zero for secant, equal to zero for tangent and less than zero for the last case.

SLOPE FORM OF TANGENT

$y = mx + c$ is always a tangent to the circle $x^2 + y^2 = a^2$ if $c^2 = a^2(1 + m^2)$. Hence, equation of tangent is, $y = mx \pm a\sqrt{1 + m^2}$ and the point of contact is $\left(-\frac{a^2 m}{c}, \frac{a^2}{c}\right)$.

POINT FORM OF TANGENT

(a) The equation of the tangent to the circle $x^2 + y^2 = a^2$ at the point (x_1, y_1) is, $xx_1 + yy_1 = a^2$.

(b) The equation of the tangent to the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ at the point (x_1, y_1) is $xx_1 + yy_1 + g(x + x_1) + f(y + y_1) + c = 0$.

In general the equation of tangent to any second degree curve at the point (x_1, y_1) on it can be obtained by replacing x^2 by xx_1, y^2 by yy_1, x by $\frac{x + x_1}{2}$, y by $\frac{y + y_1}{2}$, xy by $\frac{x_1 y + x y_1}{2}$ and c remains as c.

PARAMETRIC FORM OF TANGENT

The equation of a tangent to circle $x^2 + y^2 = a^2$ at $(a\cos\alpha, a\sin\alpha)$ is $x\cos\alpha + y\sin\alpha = a$.

The point of intersection of the tangents at the points $P(\alpha)$ & $Q(\beta)$ is $\left(\frac{a\cos\left(\frac{\alpha+\beta}{2}\right)}{\cos\left(\frac{\alpha-\beta}{2}\right)}, \frac{a\sin\left(\frac{\alpha+\beta}{2}\right)}{\cos\left(\frac{\alpha-\beta}{2}\right)}\right)$

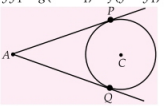
NORMAL

If a line is normal / orthogonal to a circle then it must pass through the centre of the circle. Using this fact normal to the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ at (x_1, y_1) is $(y - y_1) = \frac{y_1 + f}{x_1 + g}(x - x_1)$

PAIR OF TANGENTS FROM A POINT

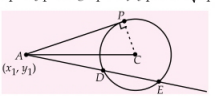
The equation of a pair of tangents drawn from the point $A(x_1, y_1)$ to the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ is $SS_1 = T^2$.

where $S \equiv x^2 + y^2 + 2gx + 2fy + c$;
$S_1 \equiv x_1^2 + y_1^2 + 2gx_1 + 2fy_1 + c$
$T \equiv xx_1 + yy_1 + g(x + x_1) + f(y + y_1) + c$.



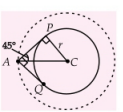
LENGTH OF A TANGENT AND POWER OF A POINT

- The length of a tangent from an external point (x_1, y_1) to the circle
$S \equiv x^2 + y^2 + 2gx + 2fy + c = 0$ is given by
$L = \sqrt{x_1^2 + y_1^2 + 2gx_1 + 2fy_1 + c} = \sqrt{S_1}$



AP = length of tangent
$AP^2 = AD \cdot AE$
Square of length of the tangent from the point P is also called the power of point w.r.t. a circle.
Power of a point w.r.t. a circle remains constant.
Power of a point P is positive, negative or zero according as the point 'P' is outside, inside or on the circle respectively.

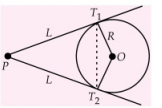
DIRECTOR CIRCLE



- The locus of the point of intersection of two perpendicular tangents is called the director circle of the given circle.
The director circle of a circle is the concentric circle having radius equal to $\sqrt{2}$ times the radius of the original circle.
$AC = r \operatorname{cosec} 45° = r\sqrt{2}$

CHORD OF CONTACT

- If two tangents PT_1 & PT_2 are drawn from the point $P(x_1, y_1)$ to the circle $S \equiv x^2 + y^2 + 2gx + 2fy + c = 0$, then the equation of the chord of contact T_1T_2 is
$xx_1 + yy_1 + g(x + x_1) + f(y + y_1) + c = 0$.
Here R = radius; L = length of tangent.
(a) Chord of contact exists only if the point 'P' is not inside.
(b) Length of chord of contact $T_1T_2 = \dfrac{2LR}{\sqrt{R^2 + L^2}}$

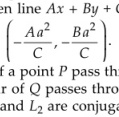


(c) Area of the triangle formed by the pair of tangents and its chord of contact = $\dfrac{RL^3}{R^2 + L^2}$
(d) Tangent of the angle between the pair of tangents from $(x_1, y_1) = \left(\dfrac{2RL}{L^2 - R^2}\right)$
(e) Equation of the circle circumscribing the triangle PT_1T_2 is
$(x - x_1)(x + g) + (y - y_1)(y + f) = 0$.

POLE AND POLAR

(a) If through a point P in the plane of the circle there be drawn any straight line to meet the circle in Q and R, the locus of the point of intersection of the tangents at Q & R is called the Polar of the point P; also P is called the Pole of the Polar.
(b) The equation to the polar of a point $P(x_1, y_1)$ w.r.t. the circle $x^2 + y^2 = a^2$ is given by $xx_1 + yy_1 = a^2$, and if the circle is general then the equation of the polar becomes $xx_1 + yy_1 + g(x + x_1) + f(y + y_1) + c = 0$ *i.e.*, $T = 0$. Note that if the point (x_1, y_1) be on the circle then the tangent and polar will be represented by the same equation. Similarly if the point (x_1, y_1) be outside the circle then the chord of contact and polar will be represented by the same equation.
(c) Pole of a given line $Ax + By + C = 0$ w.r.t. circle $x^2 + y^2 = a^2$ is $\left(-\dfrac{Aa^2}{C}, -\dfrac{Ba^2}{C}\right)$.
(d) If the polar of a point P pass through a point Q, then the polar of Q passes through P.
(e) Two lines L_1 and L_2 are conjugate of each other if pole of L_1 lies on L_2 and vice versa. Similarly two points P & Q are said to be conjugate of each other if the polar of P lies on Q and vice-versa.

EQUATION OF THE CHORD WITH A GIVEN MIDDLE POINT



The equation of the chord of the circle $S \equiv x^2 + y^2 + 2gx + 2fy + c = 0$ in terms of its midpoint $M(x_1, y_1)$ is

$xx_1 + yy_1 + g(x + x_1) + f(y + y_1) + c = x_1^2 + y_1^2 + 2gx_1 + 2fy_1 + c$ which is designated by $T = S_1$.

Remarks :

(i) The shortest chord of a circle passing through a point '*M*' inside the circle is one chord whose middle point is *M*.

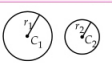
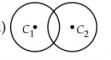
(ii) The chord passing through a point '*M*' inside the circle and which is at a maximum distance from the centre is a chord with middle point *M*.

EQUATION OF THE CHORD JOINING TWO POINTS OF CIRCLE

The equation of chord *PQ* to the circle $x^2 + y^2 = a^2$ joining two points $P(\alpha)$ and $Q(\beta)$ on it is given by the equation of a straight line joining two points α and β on the circle $x^2 + y^2 = a^2$ is

$$x\cos\left(\frac{\alpha+\beta}{2}\right) + y\sin\left(\frac{\alpha+\beta}{2}\right) = a\cos\left(\frac{\alpha-\beta}{2}\right)$$

COMMON TANGENTS OF TWO CIRCLES:

Case	Number of tangents	Condition
(i) 	4 common tangents (2 direct and 2 transverse)	$r_1 + r_2 < C_1C_2$
(ii)	3 common tangents	$r_1 + r_2 = C_1C_2$.
(iii)	2 common tangents	$\lvert r_1 - r_2 \rvert < C_1C_2 < r_1 + r_2$
(iv) 	1 common tangent.	$\lvert r_1 - r_2 \rvert = C_1C_2$
(v)	No common tangent	$C_1C_2 < \lvert r_1 - r_2 \rvert$

(Here C_1C_2 is distance between the centres of two circles.)

Remarks :

(i) The direct common tangents meet at a point which divides the line joining centre of circles externally in the ratio of their radii. Transverse common tangents meet at a point which divides the line joining centre of circles internally in the ratio of their radii.

(ii) Length of an external (or direct) common tangent and internal (or transverse) common tangent to the two circles are given by:

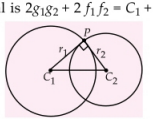
$$L_{ext} = \sqrt{d^2 - (r_1 - r_2)^2} \text{ and}$$

$$L_{int} = \sqrt{d^2 - (r_1 + r_2)^2}$$

where d = distance between the centres of the two circles and r_1, r_2 are the radii of the two circles. Note that length of internal or transverse common tangent is always less than the length of the external or direct common tangent.

ORTHOGONALITY OF TWO CIRCLES

Two circles $S_1 = 0$ and $S_2 = 0$ are said to be orthogonal or said to intersect orthogonally if the tangents at their point of intersection include a right angle. The condition for two circles to be orthogonal is $2g_1g_2 + 2f_1f_2 = C_1 + C_2$.



$(C_1C_2)^2 = (C_1P)^2 + (C_2P)^2$

$\Rightarrow (g_1 - g_2)^2 + (f_1 - f_2)^2$

$= g_1^2 + f_1^2 - C_1 + g_2^2 + f_2^2 - C_2$

$\Rightarrow 2g_1g_2 + 2f_1f_2 = C_1 + C_2$

Remarks :

(a) The centre of a variable circle orthogonal to two fixed circles lies on the radical axis of two circles.

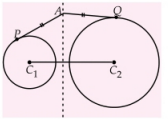
(b) If two circles are orthogonal, then the polar of a point '*P*' on first circle w.r.t. the second circle passes through the point *Q* which is the other end of the diameter through *P*. Hence locus of a point which moves such that its polars w.r.t. the circles $S_1 = 0$; $S_2 = 0$ & $S_3 = 0$ are concurrent in a circle which is orthogonal to all the three circles.

(c) The centre of a circle which is orthogonal to three given circles is the radical centre provided the radical centre lies outside all the three circles.

RADICAL AXIS AND RADICAL CENTRE

The radical axis of two circles is the locus of a point which moves such that length of tangents drawn from it on the two circles are equal. The equation of radical axis of the two circles $S_1 = 0$ & $S_2 = 0$ is given by $S_1 - S_2 = 0$

i.e. $2(g_1 - g_2)x + 2(f_1 - f_2)y + (C_1 - C_2) = 0$.



The common point of intersection of the radical axes of three circles taken two at a time is called the radical centre of three circles. Note that the length of tangents from radical centre to the three circles are equal.

- **Remarks :**
  (a) If two circles intersect, then the radical axis is the common chord of the two circles.
  (b) If two circles touch each other then the radical axis is the common tangent of the two circles at the common point of contact.
  (c) Radical axis is always perpendicular to the line joining the centres of the two circles.
  (d) Radical axis will pass through the mid point of the line joining the centres of the two circles only if the two circles have equal radii.
  (e) Radical axis bisects the direct common tangent between the two circles.
  (f) A system of circles, every two which have the same radical axis, is called a co-axial system.
  (g) Pairs of circles which do not have radical axis are concentric.

FAMILY OF CIRCLES

- The equation of the family of circles passing through the points of intersection of two circles $S_1 = 0$ and $S_2 = 0$ is : $S_1 + KS_2 = 0$
  ($K \neq -1$, provided the coefficient of x^2 & y^2 in S_1 & S_2 are same)
- The equation of the family of circles passing through the point of intersection of a circle $S = 0$ and a line $L = 0$ is given by $S + KL = 0$.
- The equation of a family of circles passing through two given points (x_1, y_1) & (x_2, y_2) can be written in the form:

$$(x-x_1)(x-x_2)+(y-y_1)(y-y_2)+K\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} = 0$$

  where K is a parameter.
- The equation of a family of circles touching a fixed line $y - y_1 = m(x - x_1)$ at the fixed point (x_1, y_1) is $(x-x_1)^2 + (y-y_1)^2 + K[y - y_1 - m(x - x_1)] = 0$, where K is a parameter.
- Family of circles circumscribing a triangle whose sides are given by $L_1 = 0$; $L_2 = 0$ and $L_3 = 0$ is given by; $L_1L_2 + \lambda L_2L_3 + \mu L_3L_1 = 0$ provided coefficient of $xy = 0$ and coefficient of x^2 = coefficient of y^2.
- Equation of circle circumscribing a quadrilateral whose sides in order are represented by the lines $L_1 = 0$, $L_2 = 0$, $L_3 = 0$ & $L_4 = 0$ are $\mu L_1L_3 + \lambda L_2L_4 = 0$ where the values of μ & λ can be found out by using condition that coefficient of x^2 = coefficient of y^2 and coefficient of $xy = 0$.

PROBLEMS

SECTION-I

Single Correct Answer Type

1. AB is a line segment of length 48 cm, C is its middle point on the line segment. AB, AC, CB semicircles are described then radius of the circle inscribed in the space enclosed by three semicircles is
(a) $3\sqrt{2}$ cm (b) 6 cm
(c) 8 cm (d) 10 cm

2. Two circles passing through the points $A(1, 2)$ and $E(2, 1)$ touch the line $4x + 8y - 7 = 0$ at B and D. The possible coordinates of point C such that $ABCD$ is a parallelogram is
(a) $\left(\frac{15}{2}, -\frac{9}{2}\right)$ (b) $(0, 3)$
(c) $\left(\frac{1}{2}, \frac{5}{2}\right)$ (d) $\left(-\frac{5}{2}, \frac{3}{4}\right)$

3. In the xy plane, the length of the shortest path from (0, 0) to (12, 16) that does not go inside the circle $(x - 6)^2 + (y - 8)^2 = 25$ is
(a) $10\sqrt{3}$ (b) $10\sqrt{5}$
(c) $10\sqrt{3} + \frac{5\pi}{3}$ (d) $10 + 5\pi$

4. A variable circle C has the equation $x^2 + y^2 - 2(t^2 - 3t + 1)x - 2(t^2 + 2t)y + t = 0$, where t is a parameter. The locus of the centre of the circle is a/an
(a) parabola (b) ellipse
(c) hyperbola (d) pair of straight lines

5. $S = 0$ and $S_1 = 0$ are two circles of radii 4 and 6 and touch externally. $L = 0$ is a direct common tangent. If p and q are radii of two circles, each of which touches externally both $S = 0$ and $S_1 = 0$ and also touches the line $L = 0$ then, harmonic mean of p and q is
(a) 1 (b) 2/5 (c) 5/6 (d) 12/5

6. If $x^2 + y^2 = 16$, $x^2 + y^2 = 36$ are two circles and P and Q move respectively on these circles such that $PQ = 4$ then the locus of mid point of PQ is a circle of radius
(a) $\sqrt{20}$ (b) $\sqrt{22}$ (c) $\sqrt{30}$ (d) $\sqrt{32}$

7. $P = \lim_{n\to\infty}\left\{\lim_{x\to 0}\dfrac{\log(1+nx)[\tanh(2x)+\tanh(4x)+\ldots+\tanh(2nx)]}{\sin^2 x+\sin^2 2x+\ldots+\sin^2 nx}\right\}$

If a circle touches x-axis at $(P, 0)$ and having y-intercept l where $l = \lim_{x\to 0}\dfrac{e^{\tan x}-e^{\sin x}}{\left(\dfrac{x}{e^{32}-1}\right)(1-\cos 2x)}$, then the radius of the circle is

(a) 1 (b) 2 (c) 5 (d) 8

8. Let x, y be real variables satisfying the equation $x^2+y^2+8x-10y-40=0$. Let $a = \max.\{(x+2)^2+(y-3)^2\}$ and $b = \min.\{(x+2)^2+(y-3)^2\}$ then

(a) $a+b=4\sqrt{2}$ (b) $a-b=18$

(c) $a+b=\sqrt{2}$ (d) $ab=73$

9. An isosceles triangle with base 24 and legs 15 each is inscribed in a circle with centre at (–1, 1), then locus of the centroid of that Δ is

(a) $4(x^2+y^2)+8x-8y-73=0$
(b) $2(x^2+y^2)+4x-4y-31=0$
(c) $2(x^2+y^2)+4x-4y-21=0$
(d) $4(x^2+y^2)+8x-8y-161=0$

10. A square of side 6 is given. C_1 is the circle circumscribing the square. A regular octagon is formed by cutting off each corner of that square. C_2 is the circle circumscribing the octagon. The locus of point of intersection of perpendicular tangents of C_1 and C_2 is a circle of radius

(a) $3\sqrt{6-2\sqrt{2}}$ (b) $2\sqrt{8-2\sqrt{2}}$

(c) $4\sqrt{\sqrt{2}-1}$ (d) $6\sqrt{3-2\sqrt{2}}$

11. If two circles $(x-1)^2+(y-3)^2=r^2$ and $x^2+y^2-8x+2y+8=0$ intersect in two distinct points, then

(a) $2<r<8$ (b) $r<2$

(c) $r>2$ (d) $r=2$

12. $P(3,2)$ is a point on the circle $x^2+y^2=13$. Two points A, B are on the circle such that $PA=PB=\sqrt{5}$ The equation of chord AB is

(a) $4x-6y+21=0$ (b) $6x+4y-21=0$
(c) $4x+6y-21=0$ (d) $6x+4y+21=0$

13. The normal of the circle $(x-2)^2+(y-1)^2=1$ which bisects the chord cut off by the line $x-2y+3=0$ is

(a) $2x+y+3=0$ (b) $2x+y-4=0$
(c) $2x+y-5=0$ (d) $2x+y-7=0$

14. The circle $x^2+y^2+4x+8y+5=0$ intersects the line $3x-4y=m$ at two distinct points if

(a) $-35<m<15$ (b) $15<m<65$
(c) $35<m<85$ (d) $-85<m<-35$

15. The length of the common chord of the circles $x^2+y^2+ax+by+c=0$ and $x^2+y^2+bx+ay+c=0$ is

(a) $\sqrt{\dfrac{(a+b)^2-8c}{2}}$ (b) $\sqrt{\dfrac{(a-b)^2-8c}{2}}$

(c) $\sqrt{\dfrac{(a-b)^2+8c}{2}}$ (d) $\sqrt{\dfrac{(a+b)^2+8c}{2}}$

16. The polars of a point P w.r.t. two given circles meet in Q. The radical axis of the circles divide PQ in the ratio

(a) 1 : 1 (b) 1 : 2 (c) 2 : 1 (d) 2 : 3

17. The radius of the least circle passing through the point (8, 4) and cutting the circle $x^2+y^2=40$ orthogonally is

(a) $\sqrt{5}$ (b) $\sqrt{7}$ (c) $2\sqrt{5}$ (d) $4\sqrt{5}$

18. The line $2x-y+1=0$ is tangent to the circle at the point (2, 5) and the centre of the circle lies on $x-2y=4$. The radius of the circle is

(a) $3\sqrt{5}$ (b) $5\sqrt{3}$ (c) $2\sqrt{5}$ (d) $5\sqrt{2}$

19. The locus of the mid points of the chords of the circle $x^2+y^2-ax-by=0$ which subtend a right angle at $\left(\dfrac{a}{2},\dfrac{b}{2}\right)$ is

(a) $ax+by=0$
(b) $ax+by=a^2+b^2$
(c) $x^2+y^2-ax-by+\dfrac{a^2+b^2}{8}=0$
(d) $x^2+y^2-ax-by-\dfrac{a^2+b^2}{8}=0$

20. A rhombus is inscribed in the region common to the two circles $x^2+y^2-4x-12=0$ and $x^2+y^2+4x-12=0$ with two of its vertices on the line joining the centres of the circles. The area of the rhombus is

(a) $8\sqrt{3}$ sq. units (b) $4\sqrt{3}$ sq. units

(c) $16\sqrt{3}$ sq. units (d) none of these

21. In a right triangle ABC, right angled at A, on the leg AC as diameter, a semicircle is described. The chord joining A with the point of intersection D of the hypotenuse and the semicircle, then the length AC equals to

(a) $\dfrac{AB\cdot AD}{\sqrt{AB^2+AD^2}}$ (b) $\dfrac{AB\cdot AD}{AB+AD}$

(c) $\sqrt{AB\cdot AD}$ (d) $\dfrac{AB\cdot AD}{\sqrt{AB^2-AD^2}}$

22. The locus of the mid points of the chords of the circle $x^2 + y^2 + 4x - 6y - 12 = 0$ which subtend an angle of $\pi/3$ radians at its circumference is

(a) $(x-2)^2 + (y+3)^2 = 6.25$
(b) $(x+2)^2 + (y-3)^2 = 6.25$
(c) $(x+2)^2 + (y-3)^2 = 18.75$
(d) $(x+2)^2 + (y+3)^2 = 18.75$

23. The angle at which the circles $(x-1)^2 + y^2 = 10$ and $x^2 + (y-2)^2 = 5$ intersect is

(a) $\pi/6$ (b) $\pi/4$ (c) $\pi/3$ (d) $\pi/2$

24. The value of 'c' for which the set,
$\{(x, y) \mid x^2 + y^2 + 2x \le 1\} \cap \{(x, y) \mid x - y + c \ge 0\}$
contains only one point in common is

(a) $(-\infty, -1] \cup [3, \infty)$ (b) $\{-1, 3\}$
(c) $\{-3\}$ (d) $\{-1\}$

25. P is a point (a, b) in the first quadrant. If the two circles which pass through P and touch both the co-ordinate axes cut at right angles, then

(a) $a^2 - 6ab + b^2 = 0$ (b) $a^2 + 2ab - b^2 = 0$
(c) $a^2 - 4ab + b^2 = 0$ (d) $a^2 - 8ab + b^2 = 0$

SECTION-II

Multiple Correct Answer Type

26. A circle passes through the points $(-1, 1)$, $(0, 6)$ and $(5, 5)$. The point(s) on this circle, the tangent(s) at which is/are parallel to the straight line joining the origin to its centre is/are

(a) $(1, -5)$ (b) $(5, 1)$
(c) $(-5, -1)$ (d) $(-1, 5)$

27. A variable circle is described to pass through the point $(1, 0)$ and tangent to the curve $y = \tan(\tan^{-1}x)$. The locus of the centre of the circle is a parabola whose

(a) length of the latusrectum is $2\sqrt{2}$
(b) axis of symmetry has the equation $x + y = 1$
(c) vertex has the coordinates $(3/4, 1/4)$
(d) none of these

28. A circle $S \equiv 0$ passes through the common points of family of circles $x^2 + y^2 + \lambda x - 4y + 3 = 0$ ($\lambda \in R$) and have minimum area then

(a) area of $S = 0$ is π sq.units
(b) radius of director circle of $S = 0$ is $\sqrt{2}$
(c) radius of director circle of $S = 0$ for x-axis is 1 unit
(d) $S = 0$ never cuts $|2x| = 1$

29. If secants are drawn through $P(2, -1)$ to the circle $x^2 + y^2 + 2x = 0$ meeting it at A and B and Q lies on $\overline{AB}$ segment then

(a) locus of Q such that PA, PQ, PB are in H.P. is $3x - y + 2 = 0$
(b) locus of Q such that PA, PQ, PB are in A.P. is $x^2 + y^2 - x + y - 2 = 0$
(c) locus of Q such that PA, PQ, PB are in G.P. is $x^2 + y^2 - 4x + 2y - 4 = 0$
(d) If a is least and b is greatest and $a \le \overline{PQ} \le b$, then G.M. of a and b is 3

30. A circle of radius 1 touches x-axis at A and y-axis at B in first quadrant. A variable line of slope 'm' through origin meets the circle at D and E

(a) length of chord $DE = \sqrt{\dfrac{8m}{1+m^2}}$
(b) If the length of chord is $\dfrac{4}{\sqrt{5}}$ the inclination can be $\tan^{-1}\left(\dfrac{1}{2}\right)$
(c) If the chord is a diameter then its slope is 1
(d) If area of ΔDEB is maximum then $m^2 = \dfrac{1}{3}$

31. The system of circles
$x^2 + y^2 + 2\lambda x - 2y + 1 = 0$ ($\lambda \in R$)

(a) possesses only one limiting point
(b) each of the member touches y-axis
(c) there are exactly two members touching x-axis
(d) the radius of the greatest circle of the system touching $y = x$ is $\sqrt{2} + 1$

32. A ray of light passing through $(-3, 1)$ reflects on x-axis at A and the reflected ray is tangential to the circle $x^2 + y^2 = 1$ at P.

(a) The reflected ray is $3x - 4y + 5 = 0$
(b) $A = \left(\dfrac{5}{3}, 0\right)$
(c) Area of ΔAPO is $\dfrac{2}{3}$ square units ('O' is origin)
(d) $P = \left(\dfrac{-3}{5}, \dfrac{4}{5}\right)$

33. If $(a, 0)$ is a point on a diameter of the circle $x^2 + y^2 = 4$ then $x^2 - 4x - a^2 = 0$ has

(a) exactly one real root in $(-1, 0]$
(b) exactly one real root in $[2, 5]$
(c) distinct roots greater than 5
(d) distinct roots greater than -1 and less than 5

34. Consider all circles through $A(2, 0)$ and $B(3, -1)$. Some of these circles will cut the circle $x^2 + y^2 - 2x + 6y - 11 = 0$ in a chord CD, then

(a) equation of CD must be of the form $-2x + 6y + \lambda = 0$
(b) equation of CD must be of the form $-3x - 5y + 17 + \lambda(x + y - 2) = 0$
(c) CD must always pass through origin

(d) CD must always pass through $\left(-\frac{7}{2}, \frac{11}{2}\right)$

35. The real numbers a and b are distinct. Consider the circles

$\omega_1 : (x-a)^2 + (y-b)^2 = a^2 + b^2$ and

$\omega_2 : (x-b)^2 + (y-a)^2 = a^2 + b^2$.

Which of the following assertions is (are) true?

(a) The line $y = x$ is an axis of symmetry for the circles.
(b) The circles intersect at the origin and a point, P(say), which lies on the line $y = x$.
(c) The line $y = x$ is the radical axis of the pair of circles.
(d) The circles are orthogonal for all $a \neq b$.

36. If two vertices of an equilateral triangle are $(-1, 0)$ and $(1, 0)$. Equation of its circumcircle is

(a) $\sqrt{3}x^2 + \sqrt{3}y^2 + 2y - \sqrt{3} = 0$

(b) $2x^2 + 2y^2 + \sqrt{3}y - 2 = 0$

(c) $\sqrt{3}x^2 + \sqrt{3}y^2 - 2y - \sqrt{3} = 0$

(d) $2x^2 + 2y^2 - \sqrt{3}y - 2 = 0$

37. No. of points of intersections of director circle of $x^2 + y^2 = 4$ and the curve

(a) $|x| + |y| = 4$ are 4 (b) $|x| + |y| = 2$ are 8
(c) $|x| + |y| = 6$ are 0
(d) $|x| + |y| = 2\sqrt{2}$ are 4

38. ω is a variable circle which passes through the point $A = (2, 3)$ and which touches the x-axis; A' is the point on ω diametrically opposite to A. The curve described by A' is a conic

(a) whose eccentricity is < 1
(b) whose latus rectum has length 12 units
(c) which has a vertex at (2, 0)
(d) which has the x-axis as a line of symmetry

SECTION-III

Comprehension Type

Paragraph for Question No. 39 to 41

To the circle $x^2 + y^2 = 4$ two tangents are drawn from $P(-4, 0)$ which touch the circle at Q and R and a rhombus $PQP'R$ is completed.

39. Circumcentre of ΔPQR is at

(a) $(-2, 0)$ (b) $(2, 0)$
(c) $\left(\frac{\sqrt{3}}{2}, 0\right)$ (d) $(1, 0)$

40. Ratio of area of $\Delta PQP'$ and $\Delta P'QR$ is

(a) 2 : 1 (b) 1 : 2 (c) $\sqrt{3} : 2$ (d) 1 : 1

41. Then area of rhombus $PQP'R$ is

(a) $6\sqrt{3}$ (b) $2\sqrt{3}$ (c) $3\sqrt{3}$ (d) $\frac{\sqrt{3}}{2}$

Paragraph for Question No. 42 to 44

If two circles touches the parabola $75y^2 = 64(5x - 3)$ at $\left(\frac{6}{5}, \frac{8}{5}\right)$ and the x-axis then

42. Centre of the smaller circle is

(a) (1, 2) (b) (2, 1) (c) (2, 2) (d) (1, 1)

43. Radius of the bigger circle is

(a) 1 (b) 2 (c) 3 (d) 4

44. The equation of smaller circle is

(a) $x^2 + y^2 - 4x - 2y + 4 = 0$
(b) $x^2 + y^2 + 4x - 2y + 2 = 0$
(c) $x^2 + y^2 - 4x - 2y - 11 = 0$
(d) $x^2 + y^2 + 4x - 2y + 4 = 0$

Paragraph for Question No. 45 to 47

The centres of the circles touching both the coordinate axes lies on the straight line $x - y = 0$ or $x + y = 0$ according to the quadrant in which the circle lies.

45. The number of circles whose centres are in the first quadrant, touching the axes and passing through a fixed point (h, k) is

(a) always be 1 (b) always be 2
(c) 1 or 2 (d) more than 2

46. If the difference between the radii of the circles passing through (a, b) and touching the axes is $c > 0$ and $ab > 0$ then the least value of $a + b$ is (relative minimum is)

(a) C (b) $\sqrt{2}C$ (c) $\frac{C}{\sqrt{2}}$ (d) $2C$

47. If one of the points of intersection of the two circles touching the axes is (p, q) then the length of their common chord is

(a) $|p \pm q|$ (b) $\sqrt{2}\,|p \pm q|$
(c) $2|p \pm q|$ (d) $\sqrt{2}\,|pq|$

Paragraph for Question No. 48 to 50

P is a variable point on the line $L = 0$. Tangents are drawn to the circle $x^2 + y^2 = 4$ from P to touch it at Q and R. The parallelogram $PQSR$ is completed.

48. If $L \equiv 2x + y = 6$, then the locus of circumcentre of ΔPQR is

(a) $2x - y = 4$ (b) $2x + y = 3$
(c) $x - 2y = 4$ (d) $x + 2y = 3$

49. If $P = (2, 3)$ then the centre of circumcircle of ΔQRS is

(a) $\left(\frac{2}{13}, \frac{7}{26}\right)$ (b) $\left(\frac{2}{13}, \frac{3}{26}\right)$

(c) $\left(\frac{3}{13}, \frac{9}{26}\right)$ (d) $\left(\frac{3}{13}, \frac{2}{13}\right)$

50. If $P = (3, 4)$ then the coordinates of S are

(a) $\left(\frac{-46}{25}, \frac{-63}{25}\right)$ (b) $\left(\frac{-51}{25}, \frac{-68}{25}\right)$

(c) $\left(\frac{-46}{25}, \frac{-68}{25}\right)$ (d) $\left(\frac{-68}{25}, \frac{-51}{25}\right)$

SECTION-IV

Matrix-Match Type

51. Match the following:

	Column-I		Column-II
(A)	The number of common tangents to the circle $x^2 + y^2 - 4x + 6y + 8 = 0$ and $x^2 + y^2 - 10x - 6y + 14 = 0$ is	(p)	1
(B)	If the circumference of the circle $x^2 + y^2 + 8x + 8y - b = 0$ is bisected by the circle $x^2 + y^2 - 2x + 4y + a = 0$ then $\frac{\lvert a+b \rvert}{14}$ equals to	(q)	2
(C)	$f(x, y) = 0$ is a circle such that $f(0, \lambda) = 0$ and $f(\lambda, 0) = 0$ have equal roots and $f(1, 1) = -2$ then the radius of the circle is	(r)	3
(D)	If a circle passes through the points of intersection of the lines $x + y + 1 = 0$ and $x + \lambda y - 3 = 0$ with the co-ordinate axes, then λ equals to	(s)	4

52. Let T_1, T_2 be two tangents drawn from $(-2, 0)$ to the circle $C : x^2 + y^2 = 1$ all circles touching C and lines T_1, T_2 are drawn

	Column-I		Column-II
(A)	Number of circles is	(p)	9
(B)	Product of the radii of all circles touching the tangents T_1 and T_2 is	(q)	1
(C)	The radius of the circle which is farthest from A is	(r)	4
(D)	Radius of a circle may be	(s)	3
		(t)	2

53. Match the following

	Column-I		Column-II
(A)	If each of 4 circles touches the 3 lines $x + 2y - 1 = 0$, $2x - 3y + 5 = 0$ and $3x + y - 1 = 0$ of radii a, b, c, d and $a > b > c > d$ then $\frac{d}{a} + \frac{d}{b} + \frac{d}{c} =$	(p)	5
(B)	If $A(-2, 0)$ and $B(2, 0)$ are two points, then the number of integral values of a if AB subtends obtuse angle at the point $(a, a + 1)$ is	(q)	2
(C)	If the lengths of the tangents from two points A & B to a circle are 3 and 4 and A and B are conjugate points then $AB =$	(r)	4
(D)	A rectangle has dimensions 4 and 2. If two circles are constructed on these sides as diameters and d is the length of common chord then $\sqrt{5}d =$	(s)	1

SECTION-V

Integer Answer Type

54. Area of part of circle $x^2 + y^2 - 4x - 6y + 12 = 0$ above the line $4x + 7y - 29 = 0$ is Δ, then $[\Delta] =$ _____, [.] is G.I.F.

55. Three points $P(a, b)$, $Q(c, d)$ and $R(e, f)$ satisfy the inequality $x^2 + y^2 - 6x - 8y < 0$ such that 'P' lies at a least distance from $A(-2, 4)$ and Q and R lies at maximum distance from A, where a, b, c, d, e and $f \in I$. The internal bisector of 'P' of ΔPQR intersects the tangents at origin and the point $\left(\frac{c+e}{2} + 1, d\right)$ to the circle $x^2 + y^2 - 6x - 8y = 0$. If the area of triangle formed by these three lines is Δ, then $\frac{3\Delta}{50}$ is equal to

56. Tangents are drawn to the circle $x^2 + y^2 = 1$ at the points where it is met by the circles $x^2 + y^2 - (\lambda + 6)x + (8 - 2\lambda)y - 3 = 0$, λ being a parameter. The locus of point of intersection of these tangents is $px + qy + 10 = 0$ then $p + q =$

57. If $r_1, r_2, r_3, \ldots$ are the radii of the several circles, each of which touches the circles $x^2 + y^2 - 8x - 18y + 93 = 0$ and $x^2 + y^2 - 2y = 3$ such that $r_1 < r_2 < r_3 < r_4 < \ldots$, then $r_2 - r_1$ equals

58. If pair of tangents to a circle in first quadrant has equation $6x^2 - 5xy + y^2 = 0$ and if one point of contact is (1, 2) and if the radius of that circle is $\sqrt{\alpha} - \sqrt{\beta}$ then $\alpha - \beta$ equals

59. For the circle $x^2 + y^2 = r^2$ the value of r for which the area enclosed by the tangents from the point $P(6, 8)$ to the circle and the chord of contact is maximum is

60. Circles are drawn through (1, 1) touching the circle $x^2 + y^2 - 6x + 2y + 1 = 0$. If r_1 and r_2 are the radii of smallest and largest circles then the value of $(r_2 + r_1)^2 - (r_2 - r_1)^2$ equals

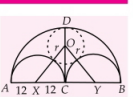
61. The line $mx - y - m - 2 = 0$ meets the circle $x^2 + y^2 - 4x + 6y - 3 = 0$ at A and B. The lines joining A and B to the centre meet the circle again at C and D. If m is a parameter and the set of lines $\overline{CD}$ are concurrent at $P(\alpha, \beta)$ then the value of $\alpha - \beta$ equals

62. From $P(-1, 1)$ tangents are drawn to $x^2 + y^2 + 2y = 0$ and A, B are points of contact. $PATB$ parallelogram is completed. If d is the distance between circumcentres of ΔPAB and ΔTAB then integral part of d is equal to

63. If the circle $x^2 + y^2 - 4x - 2y + 1 = 0$ and the line $12x - 6y - 41 = 0$ intersect at P, Q and the circle $x^2 + y^2 = 4$ and the line $x + 2y + t = 0$ intersect at R and S and if P, Q, R, S are concyclic, then the value of t equals

SOLUTIONS

1. (c) : In ΔXCO



$OX^2 = XC^2 + OC^2$

$\Rightarrow (12 + r)^2 = 12^2 + (24 - r)^2$

$\Rightarrow r = 8$

2. (a) : Equation of line AE is $x + y = 3$. Point C lies on AE such that midpoint of AC lies on BD. Let C is (α, β)

$\Rightarrow \alpha + \beta = 3$ and $4\left(\frac{\alpha+1}{2}\right) + 8\left(\frac{\beta+2}{2}\right) - 7 = 0$

$\Rightarrow \alpha = \frac{15}{2}, \beta = -\frac{9}{2}$

3. (c) : Let $O = (0, 0)$, $P = (6, 8)$ and $Q = (12, 16)$
The shortest route consists of tangent OT, minor arc TR and tangent RQ.
Since $OP = 10$, $PT = 5$, and $\angle OTP = 90°$, it follows that $\angle OPT = 60°$ and $OT = 5\sqrt{3}$

By similar reasoning, $\angle QPR = 60°$ and $QR = 5\sqrt{3}$
Because O, P and Q are collinear. $\angle RPT = 60°$, so arc TR is of length $\frac{5\pi}{3}$.

Hence the length of the shortest route is $2(5\sqrt{3}) + \frac{5\pi}{3}$

4. (a) : Centre is $x = t^2 - 3t + 1$ ... (1)
$y = t^2 + 2t$ ... (2)
(2) – (1) gives

$-x + y = 5t - 1$ or $t = \frac{1 - x + y}{5}$

Substituting the value of t in (2), we get

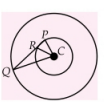
$$y = \left(\frac{y - x + 1}{5}\right)^2 + 2\left(\frac{y - x + 1}{5}\right)$$

$\Rightarrow 25y = (y - x + 1)^2 + 10(y - x + 1)$
$\Rightarrow 25y = y^2 + x^2 + 1 - 2xy - 2x + 2y + 10y - 10x + 10$
$\Rightarrow x^2 + y^2 - 2xy - 12x - 13y + 11 = 0$
which is a parabola as $\Delta \neq 0$ and $h^2 = ab$

5. (d) :
$$\left.\begin{aligned} \frac{1}{\sqrt{p}} &= \frac{1}{\sqrt{4}} + \frac{1}{\sqrt{6}} \\ \frac{1}{\sqrt{q}} &= \frac{1}{\sqrt{4}} - \frac{1}{\sqrt{6}} \end{aligned}\right\}$$

$$\text{H.M} = \frac{2}{\frac{1}{p} + \frac{1}{q}} = \frac{2}{2\left(\frac{1}{4} + \frac{1}{6}\right)} = \frac{12}{5}$$

6. (b) : Using Apollonius theorem



$CP^2 + CQ^2 = 2(CR^2 + RQ^2)$

$\Rightarrow 16 + 36 = 2\left(CR^2 + \left(\frac{4}{2}\right)^2\right)$

$\Rightarrow 52 = 2(CR^2 + 4) \Rightarrow CR = \sqrt{22}$

7. (c) : $P = \lim_{n\to\infty} \frac{n(2 + 4 + ... + 2n)}{1^2 + 2^2 + ... + n^2}$

$= \lim_{n\to\infty} \frac{n \cdot n(n+1) \times 6}{n(n+1)(2n+1)} = 3$

$I = \lim_{x\to 0} \frac{e^{\tan x} - e^{\sin x}}{e^{\tan x - \sin x}} \cdot \frac{e^{\tan x(1-\cos x)}}{(e^{x/32} - 1)(1 - \cos 2x)}$

$= 1 \times 32 \times \frac{1}{4} = 8$

Equation of circle is $x^2 + y^2 - 6x + 2fy + 9 = 0$

$\Rightarrow 2\sqrt{f^2 - 9} = 8 \Rightarrow f = \pm 5$

$\therefore$ radius = 5.

8. (d) : $x^2 + y^2 + 8x - 10y - 40 = 0$
Centre $C = (-4, 5)$, Radius $r = 9$
Let $P(2, 3)$, $CP = \pm 2\sqrt{2}$

$\therefore a = 2\sqrt{2} + 9, b = -2\sqrt{2} + 9$

$\therefore ab = 73$

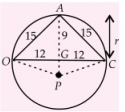
9. (d) : $PG^2 + OG^2 = OP^2$

$\Rightarrow (r - 9)^2 + 12^2 = r^2$

$\Rightarrow 81 - 18r + 144 = 0$

$\Rightarrow r = \frac{255}{18} = \frac{25}{2}$

Locus is a concentric circle with radius = PG



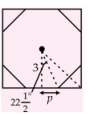
$= (r-9) + \frac{1}{3}(9) = r - 6 = \frac{25}{2} - 6 = \frac{13}{2}$

Locus is $(x+1)^2 + (y-1)^2 = \left(\frac{13}{2}\right)^2$

$\Rightarrow \quad x^2 + y^2 + 2x - 2y - \frac{161}{4} = 0$

$\Rightarrow \quad 4(x^2 + y^2) + 8x - 8y - 161 = 0$

10. (a) : Side of square = 6



$\Rightarrow \quad r_1 = \frac{6\sqrt{2}}{2} = 3\sqrt{2}$

$\tan 22\frac{1^\circ}{2} = \frac{p}{3} \Rightarrow p = 3(\sqrt{2}-1)$

$r_2 =$ radius of $C_2 = \sqrt{3^2 + (3(\sqrt{2}-1))^2}$

$= 3\sqrt{1 + (3-2\sqrt{2})} = 3\sqrt{4 - 2\sqrt{2}}$

$\therefore$ Reqd. radius

$= \sqrt{r_1^2 + r_2^2} = \sqrt{(3\sqrt{2})^2 + (3\sqrt{4-2\sqrt{2}})^2}$

$= 3\sqrt{2 + 4 - 2\sqrt{2}} = 3\sqrt{6 - 2\sqrt{2}}$

11. (a) : $|r_1 - r_2| < C_1C_2 < r_1 + r_2$

$\Rightarrow \quad |r - 3| < 5 < r + 3$

$\Rightarrow \quad 5 < r + 3 \Rightarrow 2 < r$ .....(1)

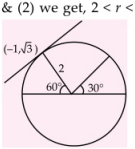
Also, $|r - 3| < 5$

$\Rightarrow \quad -5 < r - 3 < 5$

$\Rightarrow \quad -2 < r < 8$ .....(2)

from (1) & (2) we get, $2 < r < 8$

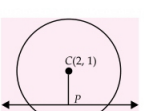
12. (b):



13. (c) : $x - 2y + 3 = 0$

P is the foot of perpendicular of C (2, 1) on $(x - 2y + 3 = 0)$

$\therefore \quad P\left(\frac{7}{5}, \frac{11}{5}\right)$



14. (a) : $r = \sqrt{15}, \; d = \frac{|-6+16-m|}{\sqrt{3^2+4^2}}$

and $r > d$

15. (a) : $2\sqrt{r^2 - d^2}$ where $r = \sqrt{\frac{a^2+b^2-4c}{4}}$

and $d = \frac{|a-b|}{2\sqrt{2}}$

16. (a) : Assume PQ as common tangent (without loss of generality). Radical axis bisects every common tangent.

17. (a) : Let the circle be $x^2 + y^2 + 2gx + 2fy + c = 0$

$\Rightarrow \quad c = 40$ and $f + 2g + 15 = 0 \Rightarrow f = -2g - 15$

Radius = $\sqrt{g^2 + (2g+15)^2 - 40}$

$g^2 + (2g + 15)^2 - 40$ to be minimum

$g = -6 \Rightarrow f = -3$.

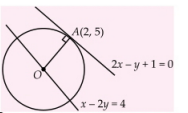
18. (a) : $2x - y + 1 = 0$ is tangent

Slope of line $OA = -1/2$

Equation of $OA \Rightarrow (y-5) = -\frac{1}{2}(x-2)$

$2y - 10 = -x + 2$ or $x + 2y = 12$

$\therefore$ Intersection of $x + 2y = 12$ with $x - 2y = 4$ will give the coordinates of centre



$x + 2y = 12$

$x - 2y = 4$

$- \quad + \quad -$

$4y = 8 \quad \Rightarrow \quad y = 2$

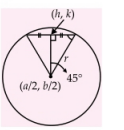
$\Rightarrow \quad x - 4 = 4 \quad \Rightarrow \quad x = 8$

$\therefore \quad C = (8, 2)$

Distance $OA = \sqrt{(8-2)^2 + (2-5)^2} = \sqrt{36+9}$

$= \sqrt{45} = 3\sqrt{5}$

19. (d): $r = \sqrt{\frac{a^2}{4} + \frac{b^2}{4}} = \frac{\sqrt{a^2+b^2}}{2}$



$\sin 45^\circ = \frac{\sqrt{\left(h - \frac{a}{2}\right)^2 + \left(k - \frac{b}{2}\right)^2}}{\frac{\sqrt{a^2+b^2}}{2}}$

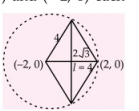
$\Rightarrow \quad \frac{1}{2} = 4\left[\frac{\frac{(2h-a)^2}{4} + \frac{(2k-b)^2}{4}}{a^2+b^2}\right]$

Simplify to get locus $x^2 + y^2 - ax - by - \frac{a^2+b^2}{8} = 0$

20. (a) : Circles with centre (2, 0) and (− 2, 0) each with radius 4

$\Rightarrow$ y axis is their common chord.

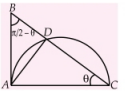
The inscribed rhombus has its diagonals equal to 4 and $4\sqrt{3}$



$\therefore \quad A = \frac{d_1 d_2}{2} = 8\sqrt{3}$ sq. units

21. (d): $\frac{AC}{AD}=\frac{1}{\sin\theta}$

$\frac{AC}{AD}=\frac{1}{\sin\theta} \Rightarrow \frac{AD}{AB}=\sin\left(\frac{\pi}{2}-\theta\right)=\cos\theta$

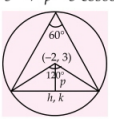


$\Rightarrow \frac{AD^2}{AC^2}+\frac{AD^2}{AB^2}=1$

Otherwise : $\Delta=\frac{1}{2}\sqrt{x^2+l^2}\cdot y=x^2l^2$

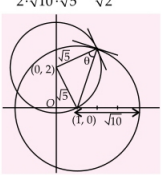
$\Rightarrow l=\frac{xy}{\sqrt{x^2-y^2}}=\frac{AB\cdot AD}{\sqrt{AB^2-AD^2}}$

22. (b): Radius = 5 $\Rightarrow$ $p = 5\cos60° = 2.5$



$\Rightarrow$ locus is $(h+2)^2+(k-3)^2=6.25$

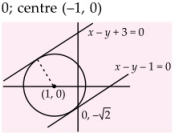
23. (b): $\cos\theta=\frac{10+5-5}{2\cdot\sqrt{10}\cdot\sqrt{5}}=\frac{1}{\sqrt{2}} \Rightarrow \theta=\frac{\pi}{4}$



24. (b): $x^2+y^2+2x-1=0$; centre $(-1, 0)$

and radius = $\sqrt{2}$

line $x - y + c = 0$



$\left|\frac{-1+c}{\sqrt{2}}\right|=\sqrt{2}$

$|c - 1| = 2$

$\Rightarrow$ $c - 1 = \pm 2$ $\Rightarrow$ $c = 3$ or -1

25. (c): Equation of the two circles be

$(x-r)^2+(y-r)^2=r^2$

i.e. $x^2+y^2-2rx-2ry+r^2=0$ where $r = r_1$ & r_2.

Condition of orthogonality gives

$2r_1r_2 + 2r_1r_2 = r_1^2 + r_2^2 \Rightarrow 4r_1r_2 = r_1^2 + r_2^2$ ....(1)

Circle passes through (a, b)

$\Rightarrow a^2+b^2-2ra-2rb+r^2=0$

i.e. $r^2-2r(a+b)+a^2+b^2=0$

$r_1 + r_2 = 2(a + b)$ and $r_1r_2 = a^2 + b^2$

Putting these values in equation (1)

$\Rightarrow a^2+b^2-4ab=0$

26. (b, d): Note that Δ is right angled at (0, 6). Centre of the circle is (2, 3). Slope of the line joining origin to the centre is 3/2. Take parametric equation of a line through (2, 3) with

$\tan\theta=-\frac{2}{3}$ as $\frac{x-2}{\cos\theta}=\frac{y-3}{\sin\theta}=\pm r$ where $r=\sqrt{13}$

Get the co-ordinates on the circle

27. (b, c): Let (h, k) be the centre, then

$\left|\frac{h-k}{\sqrt{2}}\right|=\sqrt{(h-1)^2+k^2}$

$\therefore$ $(h-k)^2=2(h^2+k^2-2h+1)$

$\Rightarrow$ $h^2+k^2+2hk-4h+2=0=2h+2k+2h-2k-2$

$\Rightarrow$ $(h+k)^2-2(h+k)+1=2(h-k-1/2)$

$\therefore$ Locus is $(x+y-1)^2=2(x-y-1/2)$

$\therefore$ Axis of symmetry is $x + y = 1$

Length of latusrectum = 2

Vertex is the point of intersection of $x + y = 1$ and $x - y = 1/2$, *i.e.* (3/4, 1/4)

28. (a, b, d): $x^2+y^2-4y+3+\lambda x=0$

$\Rightarrow$ $x = 0$ and $y^2-4y+3=0$ $\Rightarrow$ $y = 1, 3$

so common points are (0, 1) and (0, 3)

so radius of director circle is $\sqrt{2}$

Area of $S = 0$ is π sq.units

29. (a, b, c, d): (1) $s_1 = 0$

$\Rightarrow$ $x(2)+y(-1)+1(x+2)=0$ $\Rightarrow$ $3x-y+2=0$

(2) Given (2, –1), (–1, 0) as diameter ends

$\Rightarrow$ $x^2+y^2-x+y-2=0$

(3) $PQ^2 = (PA)(PB)$

$\Rightarrow$ $(x-2)^2+(y+1)^2=2^2+(-1)^2+4$

$\Rightarrow$ $x^2+y^2-4x+2y-4=0$

(4) $a=\sqrt{10}-1$, $b=\sqrt{10}+1 \Rightarrow G.M=\sqrt{10-1}=3$

30. (a, b, c, d): Given $x^2+y^2-2x-2y+1=0$ ... (i)

$y = mx$ ... (ii)

Solving $(1+m^2)x^2-2(m+1)x+1=0$

$L.C=|x_1-x_2|\sqrt{1+m^2}$

$=\frac{\sqrt{4(m+1)^2-4(m^2+1)}}{(1+m^2)}\sqrt{1+m^2}=\sqrt{\frac{8m}{1+m^2}}$

$\sqrt{\frac{8m}{1+m^2}}=\frac{4}{\sqrt{5}} \Rightarrow 16m^2+16=40m$

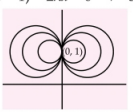
$\Rightarrow$ $2m^2-5m+2=0$ $\Rightarrow$ $m=2, \frac{1}{2}$

$f(m) = \text{Area} = \frac{1}{2} \times \text{base} \times \text{height} = \frac{1}{2} \times \sqrt{\frac{8m}{1+m^2}} \times \frac{1}{\sqrt{1+m^2}}$

Also, $f'(m) = 0 \Rightarrow m^2 = \frac{1}{3}$

31. (a, b, c, d) : Given $x^2 + y^2 + 2\lambda x - 2y + 1 = 0$

$\Rightarrow \quad (x^2 + y^2 - 2y + 1) + 2\lambda x = 0 \Rightarrow S + 2\lambda L = 0$



Clearly S & L touch $\Rightarrow$ only one L.P = (0, 1)

$x = 0$ is radical axis. Each number touches y-axis.

There are two circles touching x-axis.

Touch $y = x \Rightarrow \frac{|-\lambda - 1|}{\sqrt{2}} = |\lambda|$

$\Rightarrow \lambda^2 - 2\lambda - 1 = 0 \Rightarrow \lambda = 1 \pm \sqrt{2}$

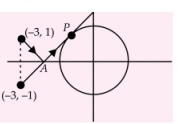
Max. radius $= 1 + \sqrt{2}$

32. (a, c, d) : $\therefore \quad y = mx + \sqrt{1+m^2}$

$\Rightarrow (3m - 1) = \sqrt{1+m^2}$

$\Rightarrow 8m^2 - 6m = 0$

$\Rightarrow m = \frac{3}{4}$



Reflected ray is

$3x - 4y - (-9 + 4) = 0 \Rightarrow 3x - 4y + 5 = 0$

Clearly $A = \left(-\frac{5}{3}, 0\right)$

$P = \left(\frac{-r^2 l}{n}, \frac{-r^2 m}{n}\right) = \left(\frac{-3}{5}, \frac{4}{5}\right)$

Area of $\Delta OAP = \frac{1}{2}\left|\left(\frac{-4}{3}\right) - 0\right| = \frac{2}{3}$

33. (a, b, d) : $-2 \le a \le 2$

Roots $= 2 + \sqrt{4 + a^2}, \; 2 - \sqrt{4 + a^2}$

34. (b, d) : The equations of circle passing through (x_1, y_1) and (x_2, y_2) is

$$(x - x_1)(x - x_2) + (y - y_2) + \lambda \begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} = 0$$

35. (a, b, c) : Interchanging x and y in one equation yields the other equation.

$\therefore$ (a) is true, since (a) is true, so is (b)

Since (b) is true, so is (c)

The centres are (a, b) and (b, a);

the radii are both $= \sqrt{a^2 + b^2}$.

The distance between the centres $= |a - b|\sqrt{2}$

The sum of the sequences of the radii

$= |a - b|\sqrt{2} = 2(a^2 + b^2)$

Now, $2(a^2 + b^2) \ne 2(a^2 - b^2)$, for all a & b

$\therefore$ (d) is false.

36. (a, c) : Since two of the vertices $A(-1, 0)$ and $B(1, 0)$ lie on x-axis, the third vertex C lies on y-axis.

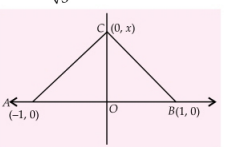
Let the coordinate of C be $(0, x)$

$(CA)^2 = (AB)^2 = (BC)^2 = 4$

$1 + x^2 = 4 \Rightarrow x = \pm\sqrt{3} \Rightarrow OC = \pm\sqrt{3}$

Since triangle is equilateral, circumcentre of the triangle coincides with centroid *i.e.* with $\left(0, \pm\frac{1}{\sqrt{3}}\right)$

Circum radius $= \frac{2}{\sqrt{3}}$



Possible equations of the circumcircle is

$x^2 + \left(y \pm \frac{1}{\sqrt{3}}\right)^2 = \left(\frac{2}{\sqrt{3}}\right)^2 \Rightarrow x^2 + y^2 \pm \frac{2}{\sqrt{3}} y - 1 = 0$

$\Rightarrow \sqrt{3}x^2 + \sqrt{3}y^2 \pm 2y - \sqrt{3} = 0$

37. (a, b, c, d) : Radius $= 2\sqrt{2}$

$|x| + |y| = 4$ touches the director circle at four points $(r = d)$

$|x| + |y| = 2\sqrt{2}$ intersects the circle where it meets the co-ordinate axes $\Rightarrow$ $4(r > d$ & points are repeated)

$|x| + |y| = 6$ does not intersect & touch the circle $(r < d)$

$|x| + |y| = 2$ intersect the circle at 8 distinct points $(r > d)$.

38. (b, c) : Let $A' = (x_1, y_1)$. The equation of ω is

$(x - x_1)(x - 2) + (y - y_1)(y - 3) = 0.$

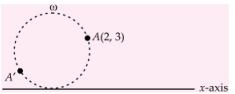
$y = 0 \Rightarrow \qquad x^2 - (x_1 + 2)x + (2x_1 + 3y_1) = 0$

This equation has equal roots.

$\Rightarrow \quad \Delta = 0 \quad \Rightarrow \quad (-(x_1 + 2))^2 = 4(2x_1 + 3y_1)$

$\Rightarrow \quad (x_1 - 2)^2 = 12y_1$

Hence A' describes the parabola $(x - 2)^2 = 12y$



39. (a) **40. (d)** **41. (a)**

QR is chord of contact $x + 1 = 0$

Circumcircle of PQR is $S + \lambda L = 0$

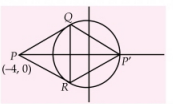
$\Rightarrow \quad x^2 + y^2 - 4 + \lambda(x + 1) = 0$

Passes through $P(-4, 0)$

$\therefore \quad \lambda = 4$

$\therefore$ Circumcentre $= (-2, 0)$

$P' = (2, 0)$ such that $PQP'R$ is a rhombus



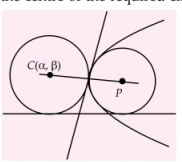
$\Delta PQP' : \Delta P'QR = 1 : 1$

Area of rhombus $PQP'R = 6\sqrt{3}$

42. (b) **43. (d)** **44. (a)**

Equation of tangent at $P\left(\frac{6}{5}, \frac{8}{5}\right)$ is $4x - 3y = 0$

Let (α, β) be the centre of the required circle



Radius of the circle $= |\beta|$

Equation of CP is $3x + 4y = 10$

$$3\alpha + 4\beta = 10 \;\Rightarrow\; \alpha = \frac{10 - 4\beta}{3}$$

$$CP^2 = P^2$$

$$\beta^2 = \left(\frac{10 - 4\beta}{3} - \frac{6}{5}\right)^2 + \left(\beta - \frac{8}{5}\right)^2$$

$$\beta^2 - 5\beta + 4 = 0$$

Centre $(-2, 4)$, $(2, 1)$; radii are 1, 4

45. (c) : Consider the circle in the first quadrant as

$x^2 + y^2 - 2rx - 2ry + r^2 = 0$

It is passes through (h, k) then

$r^2 - 2r(h + k) + (h^2 + k^2) = 0$

Discriminant, $\Delta = 4(h + k)^2 - 4(h^2 + k^2) = 8hk \geq 0$

The roots of r are real but may be equal. So 1 or 2 circles will exist.

46. (c) : As like above we get

$r^2 - 2r(a + b) + (a^2 + b^2) = 0$

We have $|r_1 - r_2| = C \;\Rightarrow\; (r_1 + r_2)^2 + 4r_1r_2 = C^2$

$\Rightarrow \quad 8ab = C^2$

$$\frac{a+b}{2} \geq \sqrt{ab} \;\Rightarrow\; a + b \geq \frac{C}{\sqrt{2}}$$

47. (b) : The distance of the point (p, q) from the line $x \pm y = 0$ is $\frac{|p \pm q|}{\sqrt{2}}$

Hence the length of the common chord is

$$2\frac{|p \pm q|}{\sqrt{2}} = \sqrt{2}\,|p \pm q|$$

48. (b) : Circumcentre of ΔPQR is the midpoint of P and centre of circle $x^2 + y^2 = 4$.

49. (c) : Find the image of cicrumcentre of ΔPQR w.r.t. chord of contact of P w.r.t. to circle.

50. (b) : Find the image of P w.r.t chord of contact of P w.r.t to circle.

51. A - r ; B - s ; C - q ; D - p

(A) $C_1 : (x - 2)^2 + (y + 3)^2 = 5$

$C_2 : (x - 5)^2 + (y - 3)^2 = 20$

Distance between centres $= \sqrt{(2-5)^2 + (3+3)^2} = 3\sqrt{5}$

$r_1 + r_2 = \sqrt{5} + 2\sqrt{5} = 3\sqrt{5}$

$\therefore$ Circle touch externally $\Rightarrow$ 3 common tangents

(B) If C_2 bisects circumference of C_1, then their common chord must be the diameter of C_1

Common chord : $10x + 4y = b + a$

Centre of C_1 lies on common chord

$$\Rightarrow \quad a + b = -40 - 16 = -56 \;\Rightarrow\; \frac{|a+b|}{14} = 4$$

52. A - r ; B - p ; C - s ; D - s

There are four circles

$$\sin\left(\frac{\theta}{2}\right) = \frac{r_2 - r_1}{r_2 + r_1} \;\Rightarrow\; \sin 30° = \frac{1 - r}{1 + r} \Rightarrow r = \frac{1}{3}$$

But radii are in G.P $\Rightarrow \quad \frac{1}{3}, 1, r'$

G.P. $\Rightarrow \quad r' = 3$

Third and fourth circles are of equal size

$$\tan 60° = \frac{r}{\sqrt{3}} \Rightarrow \sqrt{3} = \frac{r}{\sqrt{3}} \Rightarrow r = 3$$

All radii are $\frac{1}{3}, 3, 3, 3$

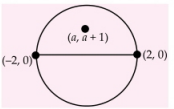
53. A - s ; B - q ; C - p ; D - r

(A) Let $a = r_1$, $b = r_2$, $c = r_3$, $d = r$

$$\frac{d}{a} + \frac{d}{b} + \frac{d}{c} = r\left(\frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3}\right) = r\left(\frac{1}{r}\right) = 1$$

(B) The point $(a, a+1)$ lies inside the circle $x^2+y^2=4$

$s_{11}<0$; $\quad a^2+(a+1)^2-4<0$



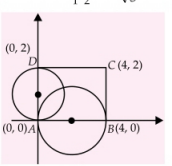
$2a^2+2a-3<0$

$a\in\left(\frac{-1-\sqrt{7}}{2},\frac{-1+\sqrt{7}}{2}\right)$

(C) If l_1, l_2 are length of tangents $AB^2=l_1^2+l_2^2$

$AB^2=9+16 \Rightarrow AB=5$

(D) Common chord $=\frac{2r_1r_2}{c_1c_2}=\frac{2\cdot2\cdot1}{\sqrt{5}}$



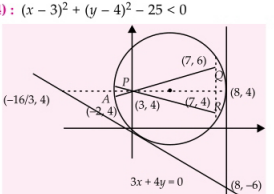
$d=\frac{4}{\sqrt{5}}$

54. (1) : Line is a diameter

$\therefore$ Required area = area of semi-circle

$\Delta=\frac{1}{2}\pi\times1=\frac{\pi}{2}$. $\quad[\Delta]=1$

55. (4) : $(x-3)^2+(y-4)^2-25<0$



Point at least distance from (–2, 4) is $P(a, b)\equiv(-1, 4)$

Point which are at greatest distance from (–2, 4) are $Q(c, d)$ and $R(e, f)\equiv Q(7, 6)$ and $R(7, 2)$

ΔPQR is an isosceles triangle and internal bisector of $\angle P$ is $y=4$

Equation of tangent at origin is $3x+4y=0$, equation of tangent at $\left(\frac{c+e}{2}+1, d\right)\equiv(8, 4)$ is $x=8$

Area of the triangle formed by the above 3 lines

$\Delta=\frac{1}{2}\times10\times\frac{40}{3}=\frac{200}{3}$. $\quad\therefore\quad 9\Delta=600$

56. (1) : $xx_1+yy_1-1=0$ ... (1)

$T_{S-S'}=0 \Rightarrow (\lambda+6)x+(2\lambda-8)y+2=0$ ... (2)

$\therefore\quad -2x_1=\lambda+6$; $\quad -2y_1=2\lambda-8$

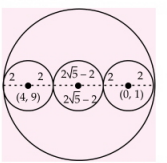
$\therefore\quad -2x_1-6=-y_1+4 \Rightarrow 2x-y+10=0$

$p=2,\ q=-1 \Rightarrow p+q=1$

57. (4) : $2r_1=\sqrt{16+64}-2-2=4\sqrt{5}-4$

$r_1=2\sqrt{5}-2$

$2r_2=(4\sqrt{5}-4)+4+4=4\sqrt{5}+4$

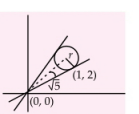


$r_2=2\sqrt{5}+2\cdot$ $\quad\therefore\quad r_2-r_1=4$

58. (5) : $\left.\begin{matrix}y=2x\\ y=3x\end{matrix}\right\}$ are tangents. $\quad\tan\theta=\frac{3-2}{1+3(1)}=\frac{1}{7}$

$\therefore\quad \tan\frac{\theta}{2}=\frac{r}{\sqrt{5}} \Rightarrow r=\sqrt{5}\cdot\tan\frac{\theta}{2}$

$=\sqrt{5}\sqrt{\frac{1-\left(\frac{7}{5\sqrt{2}}\right)}{1+\left(\frac{7}{5\sqrt{2}}\right)}}=\sqrt{5}(5\sqrt{2}-7)$



$=\sqrt{250}-\sqrt{245}$

$\therefore\quad \alpha-\beta=5$

59. (5) : $f(r)=\Delta=\frac{r.s_{11}^{3/2}}{s_{11}+r^2}=\frac{r(100-r^2)^{3/2}}{100}$

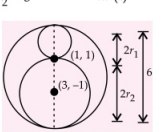
$f'(r)=0 \Rightarrow -3r^2+100-r^2=0 \Rightarrow r=5$

60. (1) : $2r_1+2r_2=6 \Rightarrow r_1+r_2=3$ ... (i)

Also $2r_2-2\sqrt{3}=3$

$\Rightarrow\quad r_2=\frac{3+2\sqrt{2}}{2}$

$\therefore\quad r_1=\frac{3-2\sqrt{2}}{2}$



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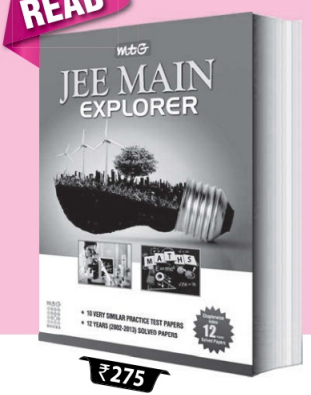

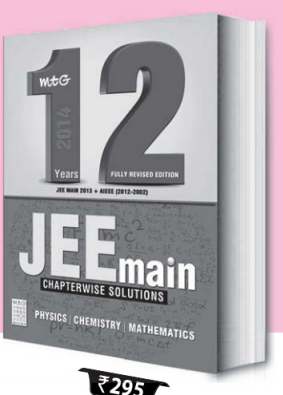

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$$G.E = 4r_1r_2 = 4 \times \frac{1}{4} = 1$$

61. (7) : Given $y + 2 = m(x - 1)$

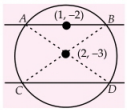
Concurrent at (1, –2)

Centre = (2, –3)

$A(x_1, y_1)$, $B(x_2, y_2)$

Then $C = (4 - x_1, -6 - y_1)$,

$D = (4 - x_2, -6 - y_2)$

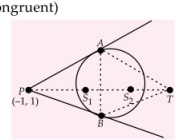


Clearly $\overline{AB} \parallel \overline{CD}$

$\overline{CD}$ concurrent at 2(2, –3) – (1, –2) = (3, –4)

$\therefore \quad \alpha - \beta = 7$

62. (1) : $PATB$ is clearly a parallelogram (ΔPAB & ΔTAB are congruent)



$\overline{AB} : S_1 = 0 \Rightarrow x - 2y - 1 = 0$

S_1 = circumcentre of

$$\Delta PAB = \frac{(-1, 1) + (0, -1)}{2} = \left(\frac{-1}{2}, 0\right)$$

d = Req. distance $= \overline{S_1S_2}$

$$= 2\left(\text{for distance from}\left(\frac{-1}{2}, 0\right) \text{to } x - 2y - 1 = 0\right)$$

$$= \frac{2\left|-\frac{1}{2} - 0 - 1\right|}{\sqrt{5}} = \frac{3}{\sqrt{5}} = 1.3$$

63. (2) : $x^2 + y^2 - 4x - 2y + 1 + \lambda(12x - 6y - 41) = 0$ ..... (1)

$x^2 + y^2 - 4 + \mu(x + 2y + t) = 0$ ..... (2)

Since 1 & 2 are same. So comparing like terms.

$\therefore \quad -4 + 12\lambda = \mu, -2 - 6\lambda = 2\mu, 1 - 41\lambda = \mu t - 4$

$$\left.\begin{aligned} \lambda &= \frac{1}{5} \\ \mu &= \frac{-8}{5} \end{aligned}\right\} \Rightarrow \quad t = 2$$

■■

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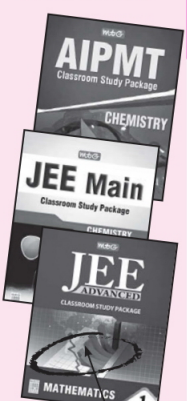

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concept BOOSTERS

Class XII

Probability

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This column is aimed at Class XII students so that they can prepare for competitive exams such as JEE Main/Advanced, etc. and be also in command of what is being covered in their school as part of NCERT syllabus. The problems here are a happy blend of the straight and the twisted, the simple and the difficult and the easy and the challenging.

CONCEPTS AND FORMULAE

- There are various phenomena in nature, leading to an outcome, which cannot be predicted apriori *e.g.* in tossing of a coin, a head or a tail may result. Probability theory aims at measuring the uncertainties of such outcomes.

ASSOCIATED TERMS

- **Random experiment :** It is a process which results in an outcome which is one of the various possible outcomes that are known to us before hand *e.g.* throwing of a die is a random experiment as it leads to fall of one of the outcome from {1, 2, 3, 4, 5, 6}. Similarly taking a card from a pack of 52 cards is also a random experiment.
- **Sample space :** It is the set of all possible outcomes of a random experiment *e.g.* {*H*, *T*} is the sample space associated with tossing of a coin.

  In set notation it can be interpreted as the universal set.

- **Event :** It is subset of sample space. *e.g.* getting a head in tossing a coin or getting a prime number in throwing a die.

  In general if a sample space consists of 'n' elements, then a maximum of 2^n events can be associated with it.
- **Complement of event :** The complement of an event 'A' with respect to a sample space S is the set of all elements of 'S' which are not in A. It is usually denoted by A', $\bar{A}$ or A^C.
- **Simple event :** If an event covers only one point of sample space, then it is called a simple event *e.g.* getting a head followed by a tail in throwing of a coin 2 times is a simple event.
- **Compound event :** When two or more than two events occur simultaneously, the event is said to be a compound event. Symbolically $A \cap B$ or AB represent the occurrence of both A & B simultaneously.

  Remark : "$A \cup B$" or $A + B$ represent the occurrence of either A or B.
- **Equally likely events :** If events have same chance of occurrence, then they are said to be equally likely. *e.g*
  - In a single toss of a fair coin, the events {*H*} and {*T*} are equally likely.
  - In a single throw of an unbiased die the events {1}, {2}, {3} and {4}, are equally likely.
  - In tossing a biased coin the events {*H*} and {*T*} are not equally likely.
- **Mutually exclusive / disjoint / incompatible events :** Two events are said to be mutually exclusive if occurrence of one of them rejects the possibility of occurrence of the other *i.e.* both cannot occur simultaneously.

  In the venn diagram the events A and B are mutually exclusive. Mathematically, we write $A \cap B = \phi$

  Events $A_1, A_2, A_3, \ldots\ldots, A_n$ are said to be mutually exclusive events iff $A_i \cap A_j = \phi \ \forall \ i, j \in \{1, 2, \ldots, n\}$ where $i \neq j$

  Remark : If $A_i \cap A_j = \phi \ \forall \ i, j \in \{1, 2, \ldots, n\}$ where $i \neq j$, then $A_1 \cap A_2 \cap A_3 \ldots \cap A_n = \phi$ but converse need not to be true.
- **Exhaustive system of events :** If each outcome of an experiment is associated with at least one of

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the events $E_1, E_2, E_3, \ldots\ldots, E_n$, then collectively the events are said to be exhaustive. Mathematically, we write

$E_1 \cup E_2 \cup E_3 \cup \ldots\ldots \cup E_n = S$. (Sample space)

CLASSICAL (A PRIORI) DEFINITION OF PROBABILITY

- If an experiment results in a total of $(m + n)$ outcomes which are equally likely and if 'm' outcomes are favourable to an event 'A' while 'n' are unfavourable, then the probability of occurrence of the event 'A', denoted by $P(A)$, is defined by

$$\frac{m}{m+n} = \frac{\text{number of favourable outcomes}}{\text{total number of outcomes}}$$

$$i.e.\quad P(A) = \frac{m}{m+n}$$

We say that odds in favour of 'A' are $m : n$, while odds against 'A' are $n : m$.

Note that $P(\bar{A})$ or $P(A')$ or $P(A^C)$, *i.e.* probability of non-occurrence of $A = \dfrac{n}{m+n} = 1 - P(A)$

In the above we shall denote the number of out comes favourable to the event A by $n(A)$ and the total number of outcomes in the sample space S by $n(S)$.

$$\therefore\quad P(A) = \frac{n(A)}{n(S)}$$

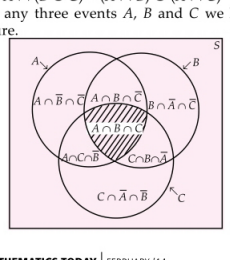
ADDITION THEOREM OF PROBABILITY

- If 'A' and 'B' are any two events associated with an experiment, then

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

- **De Morgan's laws :** If A and B are two subsets of a universal set U, then
  (a) $(A \cup B)^c = A^c \cap B^c$
  (b) $(A \cap B)^c = A^c \cup B^c$
- **Distributive laws :**
  (a) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
  (b) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

For any three events A, B and C we have the figure.



(i) $P(A \text{ or } B \text{ or } C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(C \cap A) + P(A \cap B \cap C)$

(ii) P (at least two of A, B, C occur) $= P(B \cap C) + P(C \cap A) + P(A \cap B) - 2P(A \cap B \cap C)$

(iii) P(exactly two of A, B, C occur) $= P(B \cap C) + P(C \cap A) + P(A \cap B) - 3P(A \cap B \cap C)$

(iv) P(exactly one of A, B, C occur) $= P(A) + P(B) + P(C) - 2P(B \cap C) - 2P(C \cap A) - 2P(A \cap B) + 3P(A \cap B \cap C)$

CONDITIONAL PROBABILITY

- If A and B are two events, then $P(A/B) = \dfrac{P(A \cap B)}{P(B)}$

Note that for mutually exclusive events $P(A/B) = 0$.

INDEPENDENT AND DEPENDENT EVENTS

- If two events are such that occurence or non-occurence of one does not affect the chances of occurence or non-occurence of the other event, then the events are said to be independent. Mathematically : if $P(A \cap B) = P(A)\ P(B)$, then A and B are independent.
  (i) If A and B are independent, then
    (a) A' and B' are independent,
    (b) A and B' are independent and
    (c) A' and B are independent.
  (ii) If A and B are independent, then $P(A/B) = P(A)$.
  If events are not independent, then they are said to be dependent.

INDEPENDENCY OF THREE OR MORE EVENTS

- Three events A, B and C are independent if and only if all the following conditions hold :
  $P(A \cap B) = P(A) \cdot P(B)$; $P(B \cap C) = P(B) \cdot P(C)$
  $P(C \cap A) = P(C) \cdot P(A)$;
  $P(A \cap B \cap C) = P(A) \cdot P(B) \cdot P(C)$

BINOMIAL PROBABILITY THEOREM

- If an experiment is such that the probability of success or failure does not change with trials, then the probability of getting exactly r success in n trials of such an experiment is ${}^nC_r\ p^r\ q^{n-r}$, where 'p' is the probability of a success and q is the probability of a failure in one particular experiment. Note that $p + q = 1$.

EXPECTATION

- If there are n possibilities $A_1, A_2, \ldots, A_n$ in an experiment having the probabilities $p_1, p_2, \ldots\ldots, p_n$ respectively. If value $M_1, M_2, \ldots\ldots, M_n$ are associated with the respective possibility, then the expected value of the experiment is given by $\sum_{i=1}^{n} p_i \cdot M_i$

TOTAL PROBABILITY THEOREM

- If an event A can occur with one of the n mutually exclusive and exhaustive events $B_1, B_2, \ldots, B_n$ and the probabilities $P(A/B_1)$, $P(A/B_2)$ .... $P(A/B_n)$ are known, then

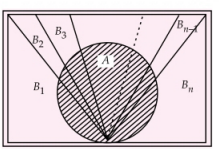
$$P(A) = \sum_{i=1}^{n} P(B_i) \cdot P(A / B_i)$$

The event A occurs with one of the n mutually exclusive and exhaustive events $B_1, B_2, B_3, \ldots, B_n$

$A = (A \cap B_1) \cup (A \cap B_2) \cup (A \cap B_3) \cup \ldots \cup (A \cap B_n)$

$P(A) = P(A \cap B_1) + P(A \cap B_2) + \ldots + P(A \cap B_n)$

$$= \sum_{i=1}^{n} P(A \cap B_i)$$



Now, $P(A \cap B_i) = P(A) \cdot P(B_i/A) = P(B_i) \cdot P(A/B_i)$

$$\therefore \quad P(A) = \sum_{i=1}^{n} P(B_i) \cdot P(A / B_i)$$

BAYES' THEOREM

- If an event A can occur with one of the n mutually exclusive and exhaustive events $B_1, B_2, \ldots, B_n$ and the probabilities $P(A/B_1)$, $P(A/B_2)$ .... $P(A/B_n)$ are known, then

$$P(B_i / A) = \frac{P(B_i) \cdot P(A / B_i)}{\sum_{i=1}^{n} P(B_i) \cdot P(A / B_i)}$$

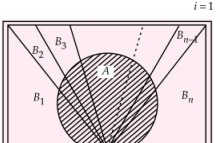
The event A occurs with one of the n mutually exclusive and exhaustive events

$B_1, B_2, B_3, \ldots\ldots, B_n$

$A = (A \cap B_1) \cup (A \cap B_2) \cup (A \cap B_3) \cup \ldots \cup (A \cap B_n)$

$P(A) = P(A \cap B_1) + P(A \cap B_2) + \ldots + P(A \cap B_n)$

$$= \sum_{i=1}^{n} P(A \cap B_i)$$



Now, $P(A \cap B_i) = P(A) \cdot P(B_i/A) = P(B_i) \cdot P(A/B_i)$

$$P(B_i / A) = \frac{P(B_i) \cdot P(A / B_i)}{P(A)} = \frac{P(B_i) \cdot P(A / B_i)}{\sum_{i=1}^{n} P(A \cap B_i)}$$

$$P(B_i / A) = \frac{P(B_i) \cdot P(A / B_i)}{\sum P(B_i) \cdot P(A / B_i)}$$

VALUE OF TESTIMONY

- If p_1 and p_2 are the probabilities of speaking the truth of two independent witnesses A and B then P(their combined statement is true)

$$= \frac{p_1 p_2}{p_1 p_2 + (1 - p_1)(1 - p_2)}$$

In this case it has been assumed that we have no knowledge of the event except the statement made by A and B.

However if p is the probability of the happening of the event before their statement, then P(their combined statement is true)

$$= \frac{p\, p_1 p_2}{p\, p_1 p_2 + (1 - p)(1 - p_1)(1 - p_2)}$$

Here it has been assumed that the statement given by all the independent witnesses can be given in two ways only, so that if all the witnesses tell falsehoods they agree in telling the same falsehood. If this is not the case and c is the chance of their coincidence testimony then the

Probability that the statement is true = $P\, p_1\, p_2$

Probability that the statement is false

$= (1 - p) \cdot c(1 - p_1)(1 - p_2)$

However chance of coincidence testimony is taken only if the joint statement is not contradicted by any witness.

PROBABILITY DISTRIBUTION

- A probability distribution spells out how a total probability of 1 is distributed over several values of a random variable.
- Mean of any probability distribution of a random variable is given by :

$$\mu = \frac{\Sigma\, p_i\, x_i}{\Sigma\, p_i} = \Sigma p_i x_i \quad (\text{Since } \Sigma\, p_i = 1)$$

- Variance of a random variable is given by,

$\sigma^2 = \Sigma (x_i - \mu)^2 \cdot p_i$

$\therefore \quad \sigma^2 = \Sigma\, p_i\, x_i^2 - \mu^2$ (Note that $SD = +\sqrt{\sigma^2}$)

- The probability distribution for a binomial variate 'X' is given by :

$P(X = r) = {}^nC_r\, p^r\, q^{n-r}$ where $P(X = r)$ is the probability of r successes.

The recurrence formula $\frac{P(r+1)}{P(r)} = \frac{n-r}{r+1} \cdot \frac{p}{q}$ is very helpful for quickly computing $P(1)\cdot P(2)\cdot P(3)$ etc, if $P(0)$ is known.

Mean of binomial probability distribution = np ; Variance of binomial probability distribution = npq.

- If p represents a person's chance of success in any venture and 'M' the sum of money which he will receive in case of success, then his expectations or probable value = pM

GEOMETRICAL APPLICATIONS

- The following statements are axiomatic :
  (i) If a point is taken at random on a given straight line segment AB, the chance that it falls on a particular segment PQ of the line segment is PQ/AB.

  i.e. probability = $\frac{\text{favourable length}}{\text{total length}}$

  (ii) If a point is taken at random on the area S which includes an area σ, the chance that the point falls on σ is σ/S. *i.e.* $\frac{\text{favourable area}}{\text{total area}}$

PROBLEMS

SECTION-I

Single Correct Answer Type

1. A and B are two weak students in Mathematics and their chances of solving a problem correctly are $\frac{1}{8}$ and $\frac{1}{12}$ respectively. They are given a problem and they obtain the same answer. If the probability of a common mistake is $\frac{1}{1001}$, then the probability that the answer was correct is

(a) $\frac{12}{13}$ (b) $\frac{13}{14}$ (c) $\frac{14}{15}$ (d) $\frac{15}{16}$

2. Sixteen players $s_1, s_2, \ldots, s_{16}$ play in a tournament. They are divided into eight pairs at random. From each pair a winner is decided on the basis of a game played between the two players of the pair. Assume that all the players are of equal strength. The probability that "exactly one of the two players s_1 & s_2 is among the eight winners" is

(a) $\frac{4}{15}$ (b) $\frac{7}{15}$ (c) $\frac{8}{15}$ (d) $\frac{9}{15}$

3. Two real numbers x, y are chosen from the interval [0, 8] then the probability that $y^2 > 2x$ is

(a) $\frac{1}{3}$ (b) $\frac{11}{32}$ (c) $\frac{11}{64}$ (d) $\frac{2}{3}$

4. Given that $x \in [0, 1]$ and $y \in [0, 1]$. Let A be the event of selecting a point (x, y) satisfying $y^2 \le x$ and B be the event selecting a point (x, y) satisfying $x^2 \le y$, then

(a) $P(A \cap B) = \frac{1}{3}$ (b) $A \subset B$

(c) $2P(A) = 3P(B)$ (d) $P(B) < P(A)$

5. An urn contains five balls. Two balls are drawn and are found to be white. The probability that all the balls are white is

(a) $\frac{1}{4}$ (b) $\frac{1}{5}$ (c) $\frac{1}{2}$ (d) $\frac{2}{5}$

6. There are four players A_1, A_2, A_3, and A_4 in pool A and four players B_1, B_2, B_3, and B_4 in pool B all players are equally capable. They are arbitrarily paired in their respective pools to play against each other and one winner is decided in each pair to play in semi final. It is known that whenever A_1 plays with A_2, A_1 wins the game and whenever B_1 plays with B_2, B_1 wins the game then the probabilities that

(i) A_1 and A_2 reach in the semi final

(ii) A_1, A_2 and B_2, B_3 reach semi final are respectively

(a) $\frac{1}{6}, \frac{1}{72}$ (b) $\frac{1}{6}, \frac{1}{6}$

(c) $\frac{1}{6}, \frac{1}{16}$ (d) none of these

7. The probability that $\sin^{-1}(\sin x) + \cos^{-1}(\cos y)$ is an integer $x, y \in \{1, 2, 3, 4\}$ is

(a) $\frac{1}{16}$ (b) $\frac{3}{16}$ (c) $\frac{15}{16}$ (d) $\frac{14}{16}$

8. A is one of 6 horses in race and is to be ridden by one of two jockeys B and C. It is 2 to 1 that B rides A, in which case all the horses are equally likely to win. If C rides A, his chance of winning is tripled. What are the odds against winning of A?

(a) 5 : 13 (b) 5 : 18 (c) 13 : 5 (d) 18 : 5

9. If $a \in [-20, 0]$, then the probability that the equation $16x^2 + 8(a + 5)x - 7a - 5 = 0$ has imaginary roots is

(a) $\frac{13}{20}$ (b) $\frac{20}{23}$ (c) $\frac{13}{24}$ (d) $\frac{15}{20}$

10. Two distinct numbers a and b are chosen randomly from the set $\{2, 2^2, 2^3, \ldots, 2^{25}\}$. Then the probability that $\log_a b$ is an integer is

(a) $\frac{131}{300}$ (b) $\frac{31}{300}$ (c) $\frac{21}{300}$ (d) $\frac{62}{300}$

11. A box has n coins, m of which are fair and the rest are biased, the probability of getting head on biased coin is 2/3. A coin is drawn from the box at random and is tossed twice. The first time it shows head and the second time it shows tail. Then the probability that the coin drawn is fair is

(a) $\frac{m}{m+8n}$ (b) $\frac{3m}{m+8n}$

(c) $\frac{9m}{m+8n}$ (d) $\frac{n}{8m+n}$

12. A number x is chosen from the set $A = \{33^n : n \in N\}$, the probability that x has 3 in the unit's place is

(a) 1/2 (b) 1/3 (c) 1/4 (d) 3/5

13. If two events A and B such that $P(A') = 0.3$, $P(B) = 0.5$ and $P(A \cap B) = 0.3$, then $P(B|A \cup B')$ is

(a) 3/8 (b) 2/3 (c) 5/6 (d) 1/4

14. A bag contains some white and some black balls, all combinations of balls being equally likely. The total number of balls in the bag is 10. If three balls are drawn at random without replacement and all of them are found to be black, the probability that the bag contains 1 white and 9 black balls is

(a) 14/55 (b) 12/55 (c) 2/11 (d) 22/55

15. There is a five-volume dictionary among 50 books arranged on a shelf in random order. The probability that these volumes stand in increasing order from left to right (the volumes are not necessarily kept side-by-side) is

(a) 1/5 (b) $1/5^{50}$ (c) $1/50^5$ (d) 1/120

16. If a and b are chosen randomly from the set consisting of numbers 1, 2, 3, 4, 5, 6 with replacement. Then the probability that $\lim_{x\to 0}[(a^x + b^x)/2]^{2/x} = 6$ is

(a) 1/3 (b) 1/4 (c) 1/9 (d) 2/9

17. Mr.A lives at origin on the Cartesian plane and has his office at (4, 5). His friend lives at (2, 3) on the same plane. Mr.A can go to his office travelling one block at a time either in the $+y$ or $+x$ direction. If all possible paths are equally likely then the probability that Mr.A passed his friends house is (shortest path for any event must be considered).

(a) 1/2 (b) 10/21 (c) 1/4 (d) 11/21

18. If three squares are selected at random from chessboard, then the probability that they form the letter 'L' is

(a) $\frac{196}{{}^{64}C_3}$ (b) $\frac{49}{{}^{64}C_3}$ (c) $\frac{36}{{}^{64}C_3}$ (d) $\frac{96}{{}^{64}C_3}$

19. If the sum of the ordinate and the abscissa of a point $P(x, y)$ is $2n$ (where x, y are natural numbers) then the probability that 'P' does not lie on the line $y = x$ is

(a) $\frac{n-1}{n+3}$ (b) $\frac{{}^{2n}C_n}{2^{2n}}$

(c) $\frac{2n-2}{2n-1}$ (d) $\frac{2n+1}{2n+3}$

20. An urn contains $(2n + 1)$ coins of which n coins have a head on both sides and the remaining $(n + 1)$ coins are fair. A coin is taken out of the urn and tossed. If the probability of obtaining a head be $\frac{37}{50}$, then n is equal to

(a) 10 (b) 11 (c) 12 (d) 13

21. A child throws 2 fair dice. If the numbers shown are unequal, he adds them together to get his final score. On the other hand, if the numbers shown are equal, he throws 2 more dice and adds all 4 numbers shown to get his final score. The probability that his final score is 4 is

(a) $\frac{1}{18}$ (b) $\frac{1}{12}$ (c) $\frac{1}{9}$ (d) $\frac{73}{1296}$

22. A die is rolled three times. The probability of getting a larger number than the previous number is

(a) $\frac{2}{7}$ (b) $\frac{5}{31}$ (c) $\frac{5}{54}$ (d) $\frac{5}{19}$

23. The probability of selecting two small squares on a chess board having a contact at corner is

(a) $\frac{7}{144}$ (b) $\frac{5}{144}$ (c) $\frac{137}{144}$ (d) $\frac{1}{16}$

24. Two numbers x and y are chosen at random without replacement from the set {1, 2, 3, 4, ..., 100}. The probability that $x^4 - y^4$ is divisible by 5 is

(a) $\frac{67}{69}$ (b) $\frac{32}{99}$ (c) $\frac{332}{495}$ (d) $\frac{134}{225}$

25. Four different objects 1, 2, 3, 4 are arranged at random on four places marked 1, 2, 3, 4. Probability that none of the objects occupies the place corresponding to the number is

(a) $\frac{3}{8}$ (b) $\frac{1}{24}$ (c) $\frac{1}{6}$ (d) $\frac{5}{8}$

26. P_1, P_2, P_3 and P_4 are four players playing in a knockout tournament and chance that P_1 will pair up with P_j is proportional to j and P_i wins if $i < j$, then probability that P_2 will reach the next round is

(a) 1/4 (b) 7/9

(c) 2/3 (d) none of these

SECTION-II

Multiple Correct Answer Type

27. Let $0 < P(A) < 1$, $0 < P(B) < 1$ and $P(A \cup B) = P(A) + P(B) - P(A)P(B)$ then which of the following is correct ?

(a) $P\left(\frac{B}{A}\right) = P(B) - P(A)$

(b) $P(\overline{A} \cup \overline{B}) = P(\overline{A}) + P(\overline{B})$

(c) $P(\overline{A \cup B}) = P(\overline{A})P(\overline{B})$

(d) $P\left(\frac{A}{B}\right) = P(A)$

28. A boy has a collection of blue and green marbles. The number of blue marbles belong to the set {2, 3, 4, ..., 13}. If two marbles are chosen simultaneously and at random from his collection, then the probability that they have different colours is 1/2. Possible number of blue marbles is

(a) 3 (b) 6 (c) 10 (d) 12

29. A and B are two independent events. The probability that A and B happen simultaneously is 1/12 and neither A nor B happens is 1/2, then

(a) $P(A) = \frac{1}{3}$, $P(B) = \frac{1}{4}$ (b) $P(A) = \frac{1}{2}$, $P(B) = \frac{1}{6}$

(c) $P(A) = \frac{1}{4}$, $P(B) = \frac{1}{3}$ (d) $P(A) = \frac{1}{6}$, $P(B) = \frac{1}{2}$

30. For two events A and B let $P(A) = 3/5$, $P(B) = 2/3$ then which of the following statements is correct?

(a) $P(A \cap \overline{B}) \le \frac{1}{3}$ (b) $P(A \cup B) \ge \frac{2}{3}$

(c) $\frac{4}{15} \le P(A \cap B) \le \frac{3}{5}$ (d) $\frac{2}{5} \le P(A/B) \le \frac{9}{10}$

31. A number x is chosen at random from the first 100 natural numbers. The probability that it satisfies

(a) $x^2 - 25x \le 150$ is 0.3

(b) $x^2 - 17x + 30 \ge 0$ is 0.88

(c) $30x - x^2$ is a perfect square of a natural number is 0.07

(d) $30x - x^2 < 0$ is 0.7

32. When a fair die is thrown twice, let (a, b) denote the outcome in which the first throw shows a and the second shows b. Further, let A, B and C be the following events : $A = \{(a, b) \mid a \text{ is odd}\}$, $B = \{(a, b) \mid b \text{ is odd}\}$ and $C = \{(a, b) \mid a + b \text{ is odd}\}$. Then

(a) $P(A \cap B) = 1/4$ (b) $P(B \cap C) = 1/4$

(c) $P(A \cap C) = 1/4$ (d) $P(A \cap B \cap C) = 0$

33. The probability that a student passes in mathematics, physics and chemistry are m, p and c respectively. Of these subjects the student has a 75% chance of passing in atleast one, a 50% chance of passing in atleast two and a 40% chance of passing exactly two. Which of the following statements is true?

(a) $p + m + c = \frac{19}{20}$ (b) $p + m + c = \frac{27}{20}$

(c) $pmc = \frac{1}{10}$ (d) $pmc = \frac{1}{4}$

34. If $\frac{1+4P}{4}, \frac{1-P}{4}, \frac{1-2P}{4}$ are probabilities of three mutually exclusive and exhaustive events, then the possible values of P belong to

(a) $\left(0, \frac{2}{3}\right)$ (b) $\left[0, \frac{1}{2}\right]$

(c) $\left[-\frac{1}{4}, \frac{1}{2}\right]$ (d) $\left[-\frac{2}{3}, \frac{2}{3}\right]$

35. A cubical die is thrown 9 times and the numbers obtained are written as a 9-digit number. The probability that the number.

(a) begins with 246 is $1/6^3$

(b) ends with 135 is $1/6^3$

(c) begins with 246 and ends with 135 is $1/6^6$

(d) begins with 246 or ends with 135 is $431/6^6$

36. Which of the following is / are true?

(a) A and B are mutually exclusive then A and B are independent.

(b) The number of 3 digit numbers abc such that $a < b < c$ is 84.

(c) The number of 3 digit numbers abc such that $a > b > c$ is 120.

(d) In a family of 3 children, A is the event that the family has children of both genders, B is the event of atmost one boy, then A and B are independent events.

37. A certain coin is tossed with probability of showing head being p. Let Q denotes the probability that when the coin is tossed four times the number of heads obtained is even. Then

(a) there is no value of p, if $Q = \frac{1}{4}$

(b) there is exactly one value of p if $Q = \frac{3}{4}$

(c) there are exactly two values of p if $Q = \frac{3}{5}$

(d) there are exactly four values of p if $Q = \frac{4}{5}$

38. A bag initially contains one red and two blue balls. An experiment consisting of selecting a ball at random, noting its colour and replacing it together with an additional ball of the same colour. If three such trials are made, then

(a) probability that atleast one blue ball is drawn is 0.9

(b) probability that exactly one blue ball is drawn is 0.2

(c) probability that all the drawn balls are red given that all the drawn balls are of same colour is 0.2

(d) probability that atleast one red ball is drawn is 0.6

39. If A and B are two events such that $P(B) \neq 1$, B^C denotes the event complementary to B, then

(a) $P(A / B^C) = \dfrac{P(A) - P(A \cap B)}{1 - P(B)}$

(b) $P(A \cap B) \geq P(A) + P(B) - 1$

(c) $P(A) > < P(A/B)$ according as $P(A/B^C) > < P(A)$

(d) $P(A/B^C) + P(A^C/B^C) = 1$

SECTION-III

Comprehension Type

Paragraph for Question Nos. 40 to 42

A is a set containing 10 elements. A subset P of A is chosen at random and set A is reconstructed by replacing the elements of P. Another subset Q of A is now chosen at random. Then the probability that if

40. $P \cup Q = A$, is

(a) $\left(\frac{1}{2}\right)^{10}$ (b) $\left(\frac{2}{3}\right)^{10}$ (c) $\left(\frac{3}{4}\right)^{10}$ (d) $\left(\frac{2}{5}\right)^{10}$

41. $P \cup Q$ contains exactly 4 elements, is

(a) $\dfrac{5805}{2^{19}}$ (b) $\dfrac{8550}{2^{19}}$ (c) $\dfrac{5058}{2^{19}}$ (d) $\dfrac{8505}{2^{19}}$

42. P and Q have no common elements, is

(a) $\left(\frac{2}{3}\right)^{10}$ (b) $\left(\frac{3}{4}\right)^{10}$ (c) $\left(\frac{4}{5}\right)^{10}$ (d) $\left(\frac{5}{6}\right)^{10}$

Paragraph for Question Nos. 43 to 45

If a particular experiment be given n(a finite) independent trials.

If the probability of success in one trial (say) p

$\therefore$ We get probability of failure, $q = (1 - p)$

The probability of r success in n trials $= {}^nC_r\, p^r q^{n-r}$

43. When two dice are thrown n number of times, the probability of getting atleast one double six is greater than 99% then the least value of n is (Given $\log 35 = 1.5441$, $\log 36 = 1.5563$)

(a) 150 (b) 154 (c) 160 (d) 164

44. Suppose the probability for A to win a game against B is 0.4. If A has an option of playing either a "best of 3 games" or a "best of five games" match against B. (No match is draw) the option A should choose so that the probability of his winning the match is higher

(a) best of 3 games (b) best of 5 games

(c) both equal (d) none of these

45. In a lot containing 100 bulbs, 10 are defective. The probability that out of a sample of 5 bulbs, none is defective is

(a) $\left(\frac{1}{10}\right)^5$ (b) $\left(\frac{1}{5}\right)^5$ (c) $\left(\frac{9}{10}\right)^5$ (d) $\frac{9}{10}$

Paragraph for Question Nos. 46 to 48

A chess match between two grandmasters X and Y is won by whoever first wins a total of two games. X's chances of winning, drawing of losing any particular game are a, b, c respectively. The games are independent and $a + b + c = 1$.

46. The probability that X wins the match after $(n + 1)$ games $(n \geq 1)$ is

(a) $na^2 n^{n-1}$

(b) $a^2(nb^{n-1} + n(n-1)b^{n-2}c)$

(c) $na^2\, bc^{n-1}$ (d) $(n-1)a^2b^{n-2}(b + nc)$

47. The probability that Y wins the match in the 4th game is

(a) $3bc^2(b + 2a)$ (b) $bc^2\,(3b + a)$

(c) $2ac^2(b + c)$ (d) $abc(2a + 3b)$

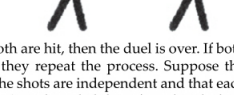
48. The probability that X wins the match is

(a) $\dfrac{a^3 + a^2c}{(a+c)^3}$ (b) $\dfrac{a^3 + 3a^2c}{(a+c)^3}$

(c) $\dfrac{a^3}{(a+c)^3}$ (d) $\dfrac{a(a^2 + 3c)}{(1-b)^3}$

Paragraph for Question Nos. 49 to 51

The commander of an batalian is punishing two of his soldiers X and Y. He arranged a duel between them. The rules of the duel are that they are to pick up their guns and shoot at each other simultaneously.



If one or both are hit, then the duel is over. If both shot miss then they repeat the process. Suppose that the results of the shots are independent and that each shot of X will hit Y with probability 2/5 and each shot of 'Y' will hit 'X' with probability 1/5.

49. The probability that 'X' is not hit is
(a) 3/25 (b) 7/25 (c) 5/13 (d) 8/13

50. The probability that both the duelists are hit is
(a) 2/13 (b) 5/13
(c) 8/13 (d) none of these

51. The probability that the duel ends after the n^{th} round of shots is

(a) $\left(\frac{12}{5}\right)^{n-1} \cdot \frac{13}{5^{n+1}}$ (b) $\left(\frac{12}{13}\right)^{n-1} \cdot \frac{13}{5^{2n}}$

(c) independent of 'n' (d) none of these

Paragraph for Question Nos. 52 to 54

Let S be a sample space and $H_1, ..., H_n$ be 'n' mutually exclusive events such that $\bigcup_{j=1}^{n} H_j = S$ and $P(H_j) > 0$ for $j = 1, 2, ..., n$. Let A be an event $P(A) > 0$. Then for $j = 1, 2, ..., n$.

$$P(H_j/A) = \frac{P(H_j)P(A/H_j)}{\sum_{k=0}^{n} P(H_k)P(A/H_k)}$$

52. In a test, an examinee either guesses or copies or knows the answer to a multiple choice question with four choices. The probability that he makes a guess is 1/3 and the probability that he copies the answer is p. The probability that his answer is correct, given that he copied it, is 1/8. If the probability that he knew the answer to question, given that he correctly answered is 8/11, then value of p is
(a) 1/2 (b) 1/3 (c) 1/6 (d) 1/12

53. Each of the n urns contains 4 white and 6 black balls. The $(n + 1)^{th}$ urn contains 5 white and 5 black balls. One of the $(n + 1)$ urns is chosen at random and two balls are drawn from it without replacement. Both the balls turn out to be black. If the probability that the $(n + 1)^{th}$ urn was chosen to draw the balls is 1/16, then value of n is
(a) 10 (b) 11 (c) 12 (d) 13

54. A letter is known to have come either from LONDON or CLIFTON. If on the postmark only two consecutive legible letters are ON, then the probability that it came from LONDON is

(a) $\frac{2}{7}$ (b) $\frac{12}{17}$ (c) $\frac{8}{9}$ (d) $\frac{7}{13}$

SECTION-IV

Matrix-Match Type

55. Five unbiased cubical dice are rolled simultaneously. Let m and n be the smallest and the largest number appearing on the upper faces of the dice, then match the probabilities given in the column II corresponding to the events given in the column I:

Column-I		Column-II	
(A)	$m = 3$	(p)	$\left(\frac{2}{3}\right)^5$
(B)	$n = 4$	(q)	$\left(\frac{2}{3}\right)^5 - \left(\frac{1}{2}\right)^4 + \left(\frac{1}{3}\right)^5$
(C)	$2 \le m \le 4$	(r)	$\left(\frac{5}{6}\right)^5 - \left(\frac{1}{3}\right)^5$
(D)	$m = 2$ and $n = 5$	(s)	$\left(\frac{2}{3}\right)^5 - \left(\frac{1}{2}\right)^5$

56. Match the data in column-I with the corresponding expressions/properties/conditions in column-II.

Column-I		Column-II	
(A)	A and B are two independent events such that $P(A) = 1/2$ and $P(B) = 1/5$	(p)	$P(A \cup B) \ge \frac{3}{5}$
(B)	A and B are two independent events such that $P(B) = 0.2$ and $P(A \cup B) = 0.6$	(q)	$P(A \mid B) = \frac{1}{2}$
(C)	A and B are two events such that $P(A) = 1/2$ and $P(B) = 2/3$	(r)	$P(A \cap B) \le \frac{1}{2}$
(D)	A and B are two independent events such that $P(A) = 3/7$ and $P(B) = 1/2$	(s)	P (Exactly one of A, B) $= \frac{1}{2}$

57. Six boys and six girls sit in a row at random. Then the probability that

Column-I		Column-II	
(A)	At least two girls sit together	(p)	$\frac{1}{462}$
(B)	No two boys sit together	(q)	$\frac{3}{22}$
(C)	Boys and girls sit alternately	(r)	$\frac{1}{132}$
(D)	Two particular boys sit together but two particular girls do not sit together is	(s)	$\frac{131}{132}$

SECTION-V

Integer Answer Type

58. A positive integer that reads the same backwards and forwards is called a palindrome. A number greater than 10 and less than 10^6 is chosen at random. If it is known that it is a five digit number and the probability that it is a palindrome is P, then the value of 300 P is equal to

59. A drawer contains several pairs of socks. Not wanting to count Mr. A asked miss X "how many pairs of socks are there in the drawer ?". Miss X, not wanting to give answer replied, "well, each sock has exactly one matching sock and the probability that two socks drawn from the drawer form a matching pair is 1/15 ". Then the answer to Mr. A's question is

60. If A and B are two events of an experiment with $P(A)=\frac{1}{4},\ P(A\cup B)=\frac{1}{2}$, then $3P\left(\frac{B}{A}\right)=$

61. One function is selected at random from all function of set $S=\{1, 2, 3, ..., n\}$ to itself. If, probability that it is one-one is $\frac{3}{32}$, then n is

62. A bag contains n identical red balls, $2n$ identical black balls and $3n$ identical white balls. If probability of drawing n balls of same colour is greater than or equal to 1/6, then minimum number of red balls in the bag is equal to

63. An experiment has 12 equally likely outcomes. Let A and B be non-empty events of the experiment. If A consists of 8 outcomes then the possible number of outcomes that B must have so that A and B are independent is

64. $P={}^{n}C_{1011},\ q={}^{n}C_{1010},\ p\neq q$ then the smallest positive integer n larger than 1010 such that p is divisible by q is a four digit number, then the sum of the digits of n is

65. A die is rolled thrice, then the probability of getting a larger number each time than the previous number is K/54. The value of "K" is

66. The papers of 4 students can be checked by any one of the 7 teachers. If the probability that all the 4 papers are checked by exactly 2 teachers is A, then the value of 49A must be

67. A coin that comes up head with probability $p>0$ and tail with probability $1-p>0$ independently on each flip, is flipped eight times. Suppose the probability of three heads and five tails is equal to $\frac{1}{25}$ of the probability of five heads and three tails. Let $p=\frac{m}{n}$, where m and n *are* relative prime positive integers. The value of $(m+n-5)$ equals

SOLUTIONS

1. (b): Let E_1 be the event of both getting the correct answer and E_2 be the event of both getting wrong answer. Let E be the event of both obtaining the same answer.

$$\therefore\quad P(E_1)=\frac{1}{8}\frac{1}{12}=\frac{1}{96},\ P(E_2)=\left(1-\frac{1}{8}\right)\left(1-\frac{1}{12}\right)=\frac{77}{96}$$

$$P\left(\frac{E}{E_1}\right)=1, P\left(\frac{E}{E_2}\right)=\frac{1}{1001}$$

$$P\left(\frac{E_1}{E}\right)=\frac{P\left(\frac{E}{E_1}\right)\cdot P(E_1)}{P\left(\frac{E}{E_1}\right)P(E_1)+P\left(\frac{E}{E_2}\right)P(E_2)}=\frac{13}{14}$$

2. (c): Req. probability $=\dfrac{{}^{2}C_1\times{}^{14}C_7}{{}^{16}C_8}=\dfrac{8}{15}$

3. (d): $n(S)=8\times 8=64.\quad \because\ x, y\in[0, 8]$

$$n(E)=\int_0^8(8-\sqrt{2}\sqrt{x})dx=\left(8x-\frac{\sqrt{2}x^{3/2}}{3/2}\right)_0^8$$

$$=64-\frac{2\sqrt{2}}{3}8\times 2\sqrt{2}=64\left(1-\frac{1}{3}\right)=64\times\frac{2}{3}$$

Required probability $=\dfrac{2}{3}$

4. (a): $P(A)=\dfrac{\int_0^1\sqrt{x}dx}{\int_0^1 dx}=\dfrac{2}{3};P(B)=\dfrac{\int_0^1\sqrt{y}dy}{\int_0^1 dy}=\dfrac{2}{3}$

$$\therefore\ P(A\cap B)=\frac{\int_0^1\sqrt{x}dx-\int_0^1 x^2dx}{\int_0^1 dx}=\frac{2}{3}-\frac{1}{3}=\frac{1}{3}$$

5. (c): $E_1=5w$, $E_2=4w$, 1 other
$E_3=3w$, 2 other ; $E_4=2w$, 3 other

$$P(A/E_1)=\frac{1}{1+\frac{6}{10}+\frac{3}{10}+\frac{1}{10}}=\frac{1}{2}$$

6. (a) : In pool A, pairing may be done in following ways :

Way number 1 : A_1A_2 and A_3A_4

Way number 2 : A_1A_3 and A_2A_4

Way number 3 : A_1A_4 and A_2A_3

Outcomes of different ways are

$W-1 \quad A_1A_3 \quad A_1A_4$

$W-2 \quad A_1A_2 \quad A_1A_4 \quad A_2A_3 \quad A_3A_4$

$W-3 \quad A_1A_2 \quad A_1A_3 \quad A_2A_4 \quad A_3A_4$

(i) The probability that A_1A_2 reach the semifinal

$$=\frac{1}{3}\left(0+\frac{1}{4}+\frac{1}{4}\right)=\frac{1}{6}$$

(ii) The probability that A_1A_2, B_1B_2 reach the semifinal

$$=\frac{1}{6}\times\frac{1}{12}=\frac{1}{72}$$

(since probability of B_2B_3 is same as that of A_2A_3 which is $\frac{1}{12}$)

7. (b) : For expression $\sin^{-1}(\sin x)+\cos^{-1}(\cos y)$ to be an integer, x should lie between $\left[\frac{\pi}{2},\frac{\pi}{2}\right]$ and y should lie between $[0, \pi]$

$x = 1$, and $y = 1, 2, 3$

$\therefore$ Required probability $= \frac{3}{16}$

8. (c) : Let E = the event that horse A wins

E_1 = the event that jockey B rides horse A

E_2 = the event that jockey C rides horse A

According to the question, the odds in favour of $E_1 = 2 : 1$

$\therefore \quad P(E_1)=\frac{2}{3}$ and $P\left(\frac{E}{E_1}\right)=\frac{1}{6}$ (since when B rides A, all six horses are equally likely to win)

$$P(E_2)=1-P(E_1)=1-\frac{2}{3}=\frac{1}{3} \;\; ; \;\; P\left(\frac{E}{E_2}\right)=3P\left(\frac{E}{E_1}\right)=\frac{1}{2}$$

Let $A_1 = E_1 \cap E$ and $A_2 = E_2 \cap E$

Now required probability $P(E) = P(A_1) + P(A_2)$

$= P(E_1 \cap E) + P(E_2 \cap E)$

$$= P(E_1)P\left(\frac{E}{E_1}\right)+P(E_2)P\left(\frac{E}{E_2}\right)=\frac{2}{3}\cdot\frac{1}{6}+\frac{1}{3}\cdot\frac{1}{2}=\frac{5}{18}$$

So that odds against winning of A are 13 : 5

9. (a) : $16x^2 + 8(a + 5)x - 7a - 5 = 0$ has imaginary root

$\Rightarrow \quad 64(a + 5)^2 + 64(7a + 5) < 0, \; -15 < a < -2$

Hence, the required probability $=\dfrac{\int_{-15}^{-2} da}{\int_{0}^{20} da}=\dfrac{[a]_{-15}^{-2}}{[a]_0^{20}}=\dfrac{13}{20}$

10. (b) : $\log_a b=\dfrac{\log b}{\log a}$

(Let $b = 2^m$ and $a = 2^n$ where m and n denotes the exponents on the base 2 in the given set = m/n)

Therefore, $\log_a b$ is an integer only if n divides m.

Now, total no. of ways m and n can be chosen

$= 25 \times 24 = 600$ for favourable cases.

Let $n = 1$. So m can take values 1, 2, 3, 4, 5, 6, ..., 24 = 24

If	$n = 2$, $m = 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24$	= 11
	$n = 3$, $m = 6, 9, 12, 15, 18, 21, 24$	= 7
	$n = 4$, $m = 8, 12, 16, 20, 24$	= 5
	$n = 5$, $m = 10, 15, 20, 25$	= 4
	$n = 6$, $m = 12, 18, 24$	= 3
	$n = 7$, $m = 14, 21$	= 2
	$n = 8$, $m = 16, 24$	= 2
	$n = 9, 10, 11, 12$, $m = 1$ for each	= 4
		62

Hence, required probability $= \frac{62}{600}=\frac{31}{300}$

11. (c) :

$$\frac{\frac{m}{n}\cdot\frac{1}{2}\cdot\frac{1}{2}}{\frac{m}{n}\cdot\frac{1}{2}\cdot\frac{1}{2}+\left(\frac{n-m}{n}\right)\frac{2}{3}\times\frac{1}{3}}$$

$$=\frac{\frac{m}{4n}}{\frac{m}{4n}+\frac{2n-2m}{9n}}=\frac{9m}{m+8n}$$

12. (c) : $n(s) = 4$, $P(E) = 1/4$

13. (a) : $P(B/A\cup B')=\dfrac{P(B\cap(A\cup B'))}{P(A\cup B')}$

$$=\frac{P(A\cap B)}{P(A)+P(B')-P(A\cap B')}=\frac{0.3}{0.7+0.5-0.4}=\frac{3}{8}$$

14. (a) : E_i = Bag has exactly i black balls $3 \le i \le 10$, $P(E_i) = 1/8$

E = Three balls drawn are black

$$P(E_9/E)=\frac{P(E_9)\,P(E/E_9)}{\sum_{i=3}^{9}P(E_i)\,P(E/E_i)}$$

$$= \frac{\frac{1}{8}\,{}^{9}C_3\,/\,{}^{10}C_3}{\frac{1}{8}\left[\frac{{}^{3}C_3}{{}^{10}C_3}+\frac{{}^{4}C_3}{{}^{10}C_3}+.....+\frac{{}^{9}C_3}{{}^{10}C_3}+\frac{{}^{10}C_3}{{}^{10}C_3}\right]}$$

$$= \frac{{}^{9}C_3}{{}^{3}C_3+{}^{4}C_3+.....+{}^{9}C_3+{}^{10}C_3} = \frac{{}^{9}C_3}{{}^{11}C_4} = \frac{14}{55}$$

15. (d): Total ways = 50!
Favourable ways = ${}^{50}C_5 \times 45!$

$$\therefore \text{ Probability} = \frac{{}^{50}C_5 \cdot 45!}{50!} = \frac{1}{120}$$

16. (c): $\lim\limits_{x\to 0}\left(\frac{a^x+b^x}{2}\right)^{2/x} = 6$

$a \in \{1 \text{ to } 6\}$, $b \in \{1 \text{ to } 6\}$
$n(S) = (1, 6)\ (2, 3)\ (3, 2)\ (6, 1) = 4$

$$\therefore \text{ Required probability } = \frac{4}{36} = \frac{1}{9}.$$

17. (b): $n(S) = \dfrac{9!}{4!\,5!} = 126$

$n(A) = 0$ to F and F to P. F is (2,3), P is (4,5)

$$= \frac{5!}{2!\,3!} \times \frac{4!}{2!\,2!} = 60$$

$$\Rightarrow \quad P(A) = \frac{60}{126} = \frac{10}{21}$$

18. (a): $n(S) = {}^{64}C_3$
Let 'E' be the event of selecting three squares which form the letter 'L'
The number of ways of selecting squares consisting of 4 unit squares is $7 \times 7 = 49$. Each square with 4 unit squares form $4L$ shapes consisting of 3 unit squares.

$$\therefore \quad n(E) = 7 \times 7 \times 4 = 196 \quad \therefore \quad \frac{n(E)}{n(S)} = \frac{196}{{}^{64}C_3}$$

19. (c): $x + y = 2n$ and $x, y \in N$
$\Rightarrow$ sample space $(S) = 2n - 1$
and out of this sample space only one point (n, n) lies on the line $y = x$. So, required probability,

$$P = \frac{(2n-1)-1}{2n-1} = \frac{2n-2}{2n-1}$$

20. (c): $\dfrac{n}{2n+1} \times 1 + \dfrac{n+1}{2n+1} \cdot \dfrac{1}{2} = \dfrac{37}{50}$

21. (d): $\dfrac{2}{36} + \dfrac{1}{6} \times \dfrac{1}{36}$

22. (c): (1, 2, 3 to 6), (1,3,4 to 6), (1,4,5 to 6), (1,5,6)
(2, 3 to 4 to 6), (2, 4, 5 to 6), (2, 5, 6)
(3, 4, 5 to 6), (3, 5, 6), (4, 5, 6)

$$P(E) = \frac{20}{216}$$

23. (a): $n(E) = 4\ (1 + 2 + 3 + 4 + 5 + 6) + 2(7) = 98$

$$P(E) = \frac{98}{{}^{64}C_2}$$

24. (a): 1, 6, 11, ..., 96
2, 7, 12, ..., 97
3, 8, 13, ..., 98
4, 9, 14, ..., 99
5, 10, 15 ....., 100
$x^4 - y^4 = (x - y)\ (x + y)\ (x^2 + y^2)$
$n(E) = 5\ ({}^{20}C_2) + 6({}^{20}C_1)({}^{20}C_1) = 3350$

25. (a): De-arrangement

$$4!\left(1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!}\right) = 9$$

$$p = \frac{9}{4!} = \frac{9}{24} = \frac{3}{8}$$

26. (b): Let the probability that P_1 will pair up with P_2, P_3 and P_4 be respectively $2k$, $3k$ and $4k$ and $2k + 3k + 4k = 1$ or $k = \dfrac{1}{9}$.

Now, P_2, will reach the next round if it is not paired with P_1.

$$\therefore \quad \text{Required probability } = 1 - \frac{2}{9} = \frac{7}{9}$$

27. (c, d): We know that
$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\therefore$ Here, $P(A \cap B) = P(A)\ P(B)$
$\Rightarrow$ A, B are independent events.
$\Rightarrow$ $A, \bar{B}; \bar{A}, B$ and $\bar{A}, \bar{B}$ are also pairwise independent events.

$$\therefore \quad P(\overline{A \cup B}) = P(\bar{A} \cap \bar{B}) = P(\bar{A})P(\bar{B})$$

$$\& \quad P(A/B) = \frac{P(A \cap B)}{P(B)} = P(A)$$

28. (a, b, c): Let the no. of blue marbles is a and no. of green marbles is b.

$$\therefore \quad \frac{ab}{{}^{a+b}C_2} = \frac{1}{2} \Rightarrow (a+b)(a+b-1) = 4ab$$

$$\Rightarrow \quad b^2 - (2a+1)b + a^2 - a = 0$$

but $b \in R \Rightarrow D = (2a+1)^2 - 4(a^2 - a) = 8a + 1$
$\therefore$ $8a + 1$ must be a perfect square.
hence possible values of a are 3,6, 10

29. (a, c) : $P(A)\cdot P(B)=\frac{1}{12}$

and $(1-P(A))(1-P(B))=\frac{1}{2}$

Solving for $P(A)$, we get $P(A)=\frac{1}{3}$ or $\frac{1}{4}$

Accordingly $P(B)=\frac{1}{4}$ or $\frac{1}{3}$

30. (a, b, c, d) : $A\cap B'\le B'\Rightarrow P(A\cap B')\le P(B')$

$A\cup B\ge B \Rightarrow P(A\cup B)\ge P(B)$

$A\cap B\le A \Rightarrow P(A\cap B)\le P(A)$

$P(A/B)=\frac{P(A\cap B)}{P(B)}$ are used

31. (a, b, d) : Total ways = 120

(a) $x^2-25x-150\le 0 \Rightarrow (x-30)(x+5)\le 0$

$\Rightarrow -5\le x\le 30$

$\text{Probability}=\frac{30}{100}=0.3$

(b) $x^2-17x+30\ge 0\Rightarrow (x-2)(x-15)\ge 0$

$\Rightarrow x\le 2$ or $x\le 15$

$\text{Probability}=\frac{88}{100}=0.88$

(c) $x(30-x)$ is a perfect square

$\Rightarrow x=3, 6, 15, 24, 27$

Probability = 0.05

(d) $x(30-x)<0 \Rightarrow x>30$

$\text{Probability}=\frac{10}{100}=0.7$

32. (a, b, c, d) : $P(A\cap B)=\frac{3}{6}\cdot\frac{3}{6}=\frac{1}{4}$

$P(B\cap C)=P(a \text{ is even and } b \text{ is odd})=\frac{1}{4}$

$P(A\cap C)=P(a \text{ is odd and } b \text{ is even})=\frac{1}{4}$

$P(A\cap B\cap C)=P(\phi)=0$

33. (b, c) : $1-(1-m)(1-p)(1-c)=\frac{3}{4}$

or, $m+p+c-mp-mc-pc+mpc=\frac{3}{4}$

$mp(1-c)+pc(1-m)+mc(1-p)+mpc=\frac{1}{2}$

$\Rightarrow mp+mc+pc-2mpc=\frac{1}{2}$

$mp(1-c)+pc(1-m)+mc(1-p)=\frac{4}{10}$

$\Rightarrow mp+mc+pc-3mpc=\frac{2}{5}$

$\therefore mpc=\frac{1}{2}-\frac{2}{5}=\frac{1}{10},\ mp+mc+pc=\frac{2}{5}+\frac{3}{10}=\frac{7}{10}$

$\therefore m+p+c=\frac{7}{10}-\frac{1}{10}+\frac{3}{4}=\frac{14-2+15}{20}=\frac{27}{20}$

34. (b, c) : We must have

$0\le\frac{1+4p}{4}\le 1,\ 0\le\frac{1-p}{4}\le 1$ and $0\le\frac{1-2p}{4}\le 1$

$\Rightarrow \frac{-1}{4}\le p\le\frac{3}{4}, -3\le p\le 1, \frac{-3}{2}\le p\le\frac{1}{2}$

Again events are mutually exclusive, so

$0\le\frac{1+4p}{4}+\frac{1-p}{4}+\frac{1-2p}{4}\le 1\Rightarrow -3\le p\le 1$

Taking intersection at all four intervals at $\frac{-1}{4}\le p\le\frac{1}{2}$.

35. (a, b, c, d) : (a) Fixing 2, 4, 6 in the first 3 places; we can fill the remaining 6 places in 6^6 ways.

Hence probability $\frac{6^6}{6^9}=\frac{1}{6^3}$

(d) Begin with 246 or end with 135

$P(E\cup F)=P(E)+P(F)-P(E\cap F)=\frac{1}{6^3}+\frac{1}{6^3}-\frac{1}{6^6}=\frac{431}{6^6}$

36. (b, c, d) : (b) : $abc,\ a<b<c$

(1, 2, 3 to 9), (1, 3, 4, to 9) (1, 4, 5 to 9), (1, 5, 6 to 9), (1, 6, 7 to 9)

(1, 7, 8 to 9), (1, 8, 9), (2, 3, 4 to 9) (2, 4, 5 to 9), (2, 5, 6 to 9), (2, 6, 7 to 9)

(2, 7, 8 to 9), (2, 8, 9) similarly remaining

Total = 84

$C: a>b>c \Rightarrow c<a<b = 84$

(+) numbers abc with $c = 0, 36$

Total = 84 + 36 = 120

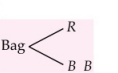
37. (a, c) : $P(0\ H \text{ or } 2\ H \text{ or } 4\ H)$

$Q=p^4+6p^2(1-p)^2-(1-p)^4=8p^4-16p^3+12p^2-4p+1$

$=\frac{(2p-1)^4+1}{2}$

38. (a, b, c, d) : (i) $P(E_1)=1-P(R\ R\ R)$

$=1-\left[\frac{1}{3}\cdot\frac{2}{4}\cdot\frac{3}{5}\right]=0.9$



(ii) $P(E_2)=3\,P(B\ R\ R)\ =3\cdot\frac{2}{3}\cdot\frac{1}{4}\cdot\frac{2}{5}=0.2$

(iii) $P(E_3)=P(R\ R\ R/R\ R\ R\cup B\ B\ B)$

$=\frac{P(RRR)}{P(RRR)+P(BBB)}$

But $P(B\ B\ B)=\frac{2}{3}\cdot\frac{3}{4}\cdot\frac{4}{5}=\frac{8}{20}\Rightarrow P(E_3)=\frac{0.1}{0.1+0.4}=0.2$

(iv) $P(E_4)=1-P(B\ B\ B)\ =1-\frac{2}{5}=0.6$

39. (a, b, c, d) : $1 \geq P(A) + P(B) - P(A \cap B)$

or $P(A \cap B) \geq P(A) + P(B) - 1$

Let $P(A) > P(A/B)$ or $P(A) > \dfrac{P(A \cap B)}{P(B)}$

$P(A) \cdot P(B) > P(A \cap B)$ .... (1)

$P(A/B^C) > P(A)$

$$\frac{P(A \cap B^C)}{P(B^C)} > P(A)$$

$P(A) - P(A \cap B) > P(A)\,[1 - P(B)]$

$- P(A \cap B) > - P(A) \cdot P(B)$

or $P(A) \cdot P(B) > P(A \cap B)$ .... (2)

From (1) and (2), $P(A) > P(A/B) \Rightarrow P(A/B^C) > P(A)$

40. (c) : $P \cup Q = A \Rightarrow n(E) = 3^{10}$ $(\because (10) \notin A)$

$$\Rightarrow P(E) = \frac{n(E)}{n(S)} = \frac{3^{10}}{4^{10}} = \left(\frac{3}{4}\right)^{10}$$

41. (d) : $n(E)$ = number of ways in which exactly 4 elements of A will belong to $P \cup Q$

$$P(E) = \frac{n(E_2)}{n(S)} = \frac{{}^{10}C_4 \cdot 3^4}{4^{10}} = \frac{8505}{2^{19}}$$

42. (b) : $\because$ $P \cap Q = \phi$, $\therefore$ $n(E) = 3^{10}$

$$\Rightarrow P(E) = \frac{3^{10}}{4^{10}} = \left(\frac{3}{4}\right)^{10}$$

43. (d) : $p = \dfrac{1}{{}^6C_1 \times {}^6C_1} = \dfrac{1}{36}$, $q = 1 - p = \dfrac{35}{36}$

As given probability of getting at least one double six $> \dfrac{99}{100}$

$$1 - \left(\frac{35}{36}\right)^n > 0.99$$

$-2 > n(\log 35 - \log 36)$ or $-2 > n(0.0122)$

$$n > \frac{2}{0.122} = 163.9$$

$\therefore$ $n > 163.9$; $n = 164$

44. (a) : Probability of A winning, say $p = 0.4 = \dfrac{4}{10}$

$\therefore$ Probability of B winning $q = 1 - 0.4 = 0.6 = \dfrac{6}{10}$

" Best of three games = (2 wins and 1 loss)

$$P_1 = {}^3C_2\left(\frac{4}{10}\right)^2\left(\frac{6}{10}\right) + {}^3C_3\left(\frac{4}{10}\right)^3 = \frac{18 \times 16 + 64}{1000} = \frac{44}{125}$$

Best of five games = (3 wins and 2 losses), (4 wins and 1 loss) or (5 wins) = $\dfrac{992}{3125}$

45. (c) : If probability of bulb being non-defective be taken as the success then $p = \dfrac{9}{10}$, $q = \dfrac{1}{10}$

46. (b) : Out of $n + 1$ matches the last match is X's win.

Out of remaining 'n' matches either one win and $n - 1$ draws or one win, one loss and $(n - 2)$ draws should be there.

$\therefore$ Probability $= {}^nC_1 \cdot a^2 b^{n-1} + (n-1) na^2 b^{n-2} c$

47. (a) : Probability that 'Y' wins a match in 4th game

$$= 3 \cdot c^2 \cdot b^2 + 6 \cdot c^2 \cdot ba = 3c^2 b(b + 2a)$$

48. (b) : Let E_n = event that 'x' wins in the n^{th} game $(n \geq 2)$

Probability that 'x' wins $= \sum_{n=2}^{\infty} P(E_n) = \sum_{n=1}^{\infty} P(E_{n+1})$

$$= \sum_{n=1}^{\infty} n \cdot a^2 b^{n-1} + (n-1) n\, a^2\, b^{n-2}\, c$$

$$= a^2 \left(\sum_{n-1}^{\infty} n\, b^{n-1} \right) + a^2 c \cdot \sum_{n=2}^{\infty} (n(n-1) b^{n-2})$$

$$= a^2 \frac{1}{(1-b)^2} + \frac{a^2 c \cdot 2}{(1-b)^3} = \frac{a^2(a+c) + 2a^2 c}{(a+c)^3} = \frac{a^3 + 3a^2 c}{(a+c)^3}$$

49. (d) : For the event that 'X' is not hit, either the game ends with X hits Y in I$^{\text{st}}$ round or both fails in I$^{\text{st}}$ and X hits Y in II$^{\text{nd}}$ or both fails in first two rounds and X hits Y in III$^{\text{rd}}$ and so on.

$\therefore$ $P(X$ is not hit)

$$= p_1 \cdot \bar{p}_2 + \bar{p}_1 \cdot \bar{p}_2 \cdot p_1 \cdot \bar{p}_2 + (\bar{p}_1)^2 \cdot (\bar{p}_2)^2 \cdot p_1 \cdot \bar{p}_2 + \ldots \infty$$

$$= \frac{2}{5} \cdot \frac{4}{5} + \frac{3}{5} \cdot \frac{4}{5} \cdot \frac{2}{5} \cdot \frac{4}{5} + \left(\frac{3}{5}\right)^2 \cdot \left(\frac{4}{5}\right)^2 \cdot \frac{2}{5} \cdot \frac{4}{5} \ldots \infty$$

$$= \frac{\frac{2}{5} \cdot \frac{4}{5}}{1 - \frac{3}{5} \cdot \frac{4}{5}} = \frac{8}{13}$$

50. (a) : P(both the duelists are hit)

$$= p_1 \cdot p_2 + \bar{p}_1 \cdot \bar{p}_2 \cdot p_1 \cdot p_2 + (\bar{p}_1)^2 \cdot (\bar{p}_2)^2 \cdot p_1 \cdot p_2 + \ldots \infty$$

$$= \frac{p_1 \cdot p_2}{1 - p_1 \cdot p_2} = \frac{\frac{2}{5} \cdot \frac{1}{5}}{1 - \frac{3}{5} \cdot \frac{4}{5}} = \frac{2}{13}$$

51. (a) : For the event that duel ends after the n^{th} round of shots, both duelists should fail till $(n-1)^{\text{th}}$ round and in n^{th} round at least one of them should be hitted.

$\therefore$ P(duel ends after the n^{th} round)

$= (\overline{p}_1 \cdot \overline{p}_2)^{n-1}(p_1 \cdot p_2 + p_1 \cdot \overline{p}_2 + \overline{p}_1 \cdot p_2)$

$= \left(\frac{3}{5}\cdot\frac{4}{5}\right)^{n-1}\left(\frac{2}{5}\cdot\frac{1}{5}+\frac{2}{5}\cdot\frac{4}{5}+\frac{3}{5}\cdot\frac{1}{5}\right) = \left(\frac{12}{5}\right)^{n-1}\cdot\frac{13}{5^{n+1}}$

52. (b): Let G denote the event that examinee guesses, C that he copies; K that he knows the answer and R that his answer is right.

We have, $P(G) = 1/3$, $P(C) = p$ and

$p(K) = 1 - 1/3 - p = 2/3 - p$

Also, $P(R/G) = 1/4$ [out of 4 choices only one is correct]

$P(R/C) = 1/8$ [given] and $P(R/K) = 1$.

By the Baye's theorem

$$P(K/R) = \frac{P(K)P(R/K)}{P(G)P(R/G)+P(C)P(R/C)+P(K)P(R/K)}$$

$$= \frac{(2/3-p)(1)}{(1/3)(1/4)+p(1/8)+(2/3-p)(1)} = \frac{16-24p}{18-21p}$$

According to the given condition,

$$\frac{16-24p}{18-21p} = \frac{8}{11} \Rightarrow \frac{2-3p}{18-21p} = \frac{1}{11} \text{ or } p = \frac{1}{3}$$

53. (a): Let E_1, E_2 denotes the event that one of the first n urns $(n+1)^{\text{th}}$ urn is chosen and A denote the event that two balls drawn without replacement from the urn are black. We have,

$$P(E_1) = \frac{n}{n+1},\ P(E_2) = \frac{1}{n+1}$$

$$P(A/E_1) = \frac{{}^6C_2}{{}^{10}C_2} = \frac{1}{3} \text{ and } P(A/E_2) = \frac{{}^5C_2}{{}^{10}C_2} = \frac{2}{9}$$

By the Baye's rule,

$$P(E_2/A) = \frac{P(E_2)P(A/E_2)}{P(E_1)P(A/E_1)+P(E_2)P(A/E_2)}$$

$$\Rightarrow \frac{1}{16} = \frac{\left(\frac{1}{n+1}\right)\left(\frac{2}{9}\right)}{\left(\frac{n}{n+1}\right)\left(\frac{1}{3}\right)+\left(\frac{1}{n+1}\right)\left(\frac{2}{9}\right)} = \frac{2}{3n+2} \Rightarrow n = 10$$

54. (b): Let

A_1 : selecting a pair of consecutive letters from the word LONDON

A_2 : Selecting a pair of consecutive letters from the word CLIFTON.

E : Selecting a pair of letters 'ON'

$$P\left(\frac{A_1}{E}\right) = \frac{P(A_1)P\left(\frac{E}{A_1}\right)}{P(A_1)P\left(\frac{E}{A_1}\right)+P(A_2)P\left(\frac{E}{A_2}\right)} = \frac{12}{17}$$

55. A - s; B - s; C - r; D - q

(A) The number appearing on upper face of any dice can be 3, 4, 5 or 6 *i.e.* maximum 4 cases.

$$P(m=3) = P(m\geq 3) - P(m\geq 4) = \frac{4^5-3^5}{6^5} = \left(\frac{2}{3}\right)^5 - \left(\frac{1}{2}\right)^5$$

(B) The number appearing on upper face of any dice can be 1, 2, 3, or 4 *i.e.* maximum 4 cases.

$$P(n=4) = P(n\leq 4) - P(n\leq 3) = \frac{4^5-3^5}{6^5} = \left(\frac{2}{3}\right)^5 - \left(\frac{1}{2}\right)^5$$

(C) $P(2\leq m\leq 4) = P(m\geq 2) - P(m\geq 5)$

$$= \frac{5^5-2^5}{6^5} = \left(\frac{5}{6}\right)^5 - \left(\frac{1}{3}\right)^5$$

(D) $P(m=2, n=5)$

$= P(2, 3, 4 \text{ or } 5) - P(2, 3 \text{ or } 4) - P(3, 4 \text{ or } 5) + P(3 \text{ or } 4)$

$$= \frac{4^5-2\times 3^5+2^5}{6^5} = \left(\frac{2}{3}\right)^5 - \left(\frac{1}{2}\right)^4 + \left(\frac{1}{3}\right)^5$$

56. A - (p, q, r, s); B - (p, q, r, s); C - (p, r) ; D - (r, s)

(A) $P(A\cup B) = 1 - P(A'\cap B') = 1 - \frac{1}{2}\times\frac{4}{5} = \frac{3}{5}$

$P(A/B) = P(A) = \frac{1}{2}$ as A and B are independent

$P(A\cap B) \leq \min\{P(A),\ P(B)\} = \frac{1}{5} \leq \frac{1}{2}$

P(Exactly of A, B) $= P(A\cap B') + P(A'\cap B)$

$= P(A)\ P(B') + P(A')\ P(B) = \frac{1}{2}\times\frac{4}{5} + \frac{1}{2}\times\frac{1}{5} = \frac{1}{2}$

(B) $P(A\cup B) = P(A) + P(B) - P(A\cap B)$

$0.6 = P(A) + 0.2 - 0.2\ P(A)$

$0.4 = 0.8\ P(A) \Rightarrow P(A) = \frac{1}{2}$

$\therefore$ $P(A/B) = \frac{1}{2}$ as A and B are independent

Also $P(A\cap B) = P(A)P(B) = 0.1 \leq \frac{1}{2}$

So, P(Exactly one of A, B) $= \frac{1}{2}$

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(C) $P(A \cup B) \geq \max\{P(A), P(B)\} = \frac{2}{3} \geq \frac{3}{5}$

$P(A \cap B) \leq \max\{P(A), P(B)\} = \frac{1}{2}$

(D) $P(A \cup B) \geq \frac{1}{2}$, $P(A \cap B) \leq \frac{3}{7} \leq \frac{1}{2}$

$P(A/B) = \frac{3}{7}$ and P(exactly one of A, B) $= \frac{1}{2}$

57. A - s; B - r ; C - p; D - q

Total ways = 12!

(A) $1 - \frac{6! \times {}^7P_6}{12!} - 1 - \frac{1}{132} = \frac{131}{132}$

(B) Probability $= \frac{6! \times {}^7P_6}{12!} = \frac{1}{132}$

(C) $\frac{2 \cdot 6!\ 6!}{12!} = \frac{1}{462}$

(D) Treat the two particular boys are one unit. Then 5-boys and 4-girls can be arranged in 9! units. Now the two particular girls can be inserted in 10 gaps in ${}^{10}P_2$ ways and the two particular boys can be inter changed in 2 ways.

$\therefore$ Probability $= \frac{9! \times 2 \times {}^{10}P_2}{12!} = \frac{3}{22}$

58. (3) : Let A be the event of choosing 5-digit no. and E be the event the number is a palindrome (*abcba*).

$\therefore \quad n(A) = 9(10^4) = 90000$

and $n(A \cap B) = 9 \times 10^2 = 900$

$\therefore \quad P\left(\frac{E}{A}\right) = \frac{n(E \cap A)}{n(A)} = \frac{1}{100} = P$

59. (8) : Let the number of socks be $2n$ forming n-pairs of socks.

$\therefore \quad n(S) = {}^{2n}C_2$, $n(A) = {}^nC_1$ (no. of ways of chosing exaetly one pair out of n pairs)

$\therefore \quad P(A) = \frac{{}^nC_1}{{}^{2n}C_2} = \frac{1}{15} \Rightarrow 2n - 1 = 15 \Rightarrow n = 8$

60. (1) : $P\left(\frac{B}{\bar{A}}\right) = \frac{P(B \cap \bar{A})}{P(\bar{A})} = \frac{P(A \cup B) - P(A)}{1 - P(A)}$

$$= \frac{\frac{1}{2} - \frac{1}{4}}{1 - \frac{1}{4}} = \frac{\frac{1}{4}}{\frac{3}{4}} = \frac{1}{3}$$

61. (4) : $\frac{n!}{n^n} = \frac{3}{32} = \frac{8 \times 3}{8 \times 32} = \frac{24}{256} = \frac{4!}{4^4} \Rightarrow n = 4$

62. (5) : Total no. of cases = coefficient of x^n in $(1 + x + x^2 + ... + x^n)^3 = {}^{n+2}C_2$

So required probability $= \frac{3}{{}^{n+2}C_2} = \frac{6}{(n+1)(n+2)} \geq \frac{1}{6}$

$\Rightarrow \quad n \geq \frac{-3 + \sqrt{145}}{2}$; $n_{\min} = 5$

63. (4) : Let $n(B) = x$, $n(A \cap B) = y$

$\Rightarrow \quad \frac{8}{12} \cdot \frac{x}{12} = \frac{y}{12} \Rightarrow 8x = 12y$

$\Rightarrow \quad x = 3, 6, 9$ or 12

64. (8) : $\frac{p}{q} = \frac{n - 1010}{1011} = k$ (say)

Thus $n = 1011\,k + 1010$

For $k = 2 \Rightarrow n = 3032$

65. (5) : $\sum_{i=2}^{5} (i-1) \cdot (6-i) = 1 \cdot 4 + 2 \cdot 3 + 3 \cdot 2 + 4 \cdot 1 = 20$

$p = \frac{20}{216} = \frac{5}{54}$

66. (6) : Total number of ways in which papers of 4 students, can be checked by seven teachers = 7^4

Now, choosing two teachers out of 7 is ${}^7C_2 = 21$

The number of ways = (21) (14)

Required probability $= \frac{(21)(14)}{7^4} = \frac{6}{49} = A$

67. (6) : We have

$${}^8C_3\, p^3(1-p)^5 = \frac{1}{25}\, {}^8C_5\, p^5(1-p)^3$$

or $1 - p = \frac{(1)(p)}{5}$

So we find $p = \frac{5}{6} \Rightarrow m + n - 5 = 6$

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Mock Test PAPER

West Bengal JEE 2014

The entire syllabus of Mathematics of WB-JEE is being divided into six modules, in each module there will be a Mock Test Paper (MTP) and two MTP in the whole syllabus which will be published in subsequent issues.
The syllabus for module break-up is given below :

Module I : *Algebra - I,* **Module - II :** *Trigonometry,* **Module - III :** *Co-ordinate geometry of two dimensions,*
Module - IV : *Differential calculus and its applications,* **Module - V :** *Algebra - II,* **Module - VI :** *Integral calculus and its applications, Differential equations.*

Module – V : ALGEBRA-II

CONTENTS :

Principle of Mathematical Induction : Statement of the principle. Proof by induction for the sum of squares, sum of cubes of first n natural numbers, divisibility properties like $2^{2n} - 1$ is divisible by 3 ($n \geq 1$), 7 divides $3^{2n} + 1 + 2^{n+2}$($n \geq 1$).

Binomial theorem (positive integral index) : Statement of the theorem, general term, middle term, equidistant terms, properties of binomial co-efficients.

Infinite series : Binomial theorem for negative and fractional index. Infinite G.P. series, Exponential and Logarithmic series with range of validity (statement only), simple applications.

Matrices : Concepts of $m \times n$ ($m \leq 3$, $n \leq 3$) real matrices, operations of addition, scalar multiplication and multiplication of matrices. Transpose of a matrix. Determinant of a square matrix. Properties of determinants (statement only). Minor, cofactor and adjoint of a matrix. Non-singular matrix. Inverse of a matrix. Finding area of a triangle. Solutions of system of linear equations. (Not more than 3 variables).

Probability : Classical definition, addition rule, conditional probability and Baye's theorem, independence, multiplication rule.

This paper is useful for JEE Main & Advanced and other Engineering Entrance Exams.

MODULE - V : Algebra-II

Time : 45 mins | Marks : 30

Question numbers 1 to 24, carries 1 mark each.

1. If $P = \begin{bmatrix} 1 & \alpha & 3 \\ 1 & 3 & 3 \\ 2 & 4 & 4 \end{bmatrix}$ is adjoint of a 3×3 matrix A and $|A| = 4$, then α is equal to
(a) 11 (b) 5 (c) 0 (d) 4

2. The term independent of x in expansion of $\left(\frac{x+1}{x^{\frac{2}{3}} - x^{\frac{1}{3}} + 1} - \frac{x-1}{x - x^{\frac{1}{2}}} \right)^{10}$ is
(a) 120 (b) 210 (c) 310 (d) 4

3. If $A = \begin{vmatrix} x & 1 & 1 \\ 1 & x & 1 \\ 1 & 1 & x \end{vmatrix}$ and $B = \begin{vmatrix} x & 1 \\ 1 & x \end{vmatrix}$, then $\frac{dA}{dx} =$
(a) $3B + 1$ (b) $3B$ (c) $-3B$ (d) $1 - 3B$

4. The value of $C_0 + 2C_1 + 3C_2 + \ldots + (n+1) \cdot C_n = 576$, then n is
(a) 7 (b) 5 (c) 6 (d) 9

5. The remainder when, $10^{10} \cdot (10^{10} + 1)(10^{10} + 2)$ is divided by 6 is
(a) 2 (b) 4 (c) 0 (d) 6

6. If $(1 + x + x^2)^n = 1 + a_1 x + a_2 x^2 + \ldots + a_{2n} x^{2n}$, then $2a_1 - 3a_2 + \ldots - (2n+1)\, a_{2n} =$
(a) n (b) $-n$ (c) $n + 1$ (d) $-n - 1$

7. The value of x satisfying the equation $\begin{vmatrix} \cos 2x & \sin 2x & \sin 2x \\ \sin 2x & \cos 2x & \sin 2x \\ \sin 2x & \sin 2x & \cos 2x \end{vmatrix} = 0$ and $x \in \left[0, \frac{\pi}{4}\right]$ is

By : Sankar Ghosh, HOD(Math), Takshyashila. Mob : 09831244397.

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{2}$ (c) $\frac{\pi}{16}$ (d) $\frac{\pi}{8}$

8. If t_5, t_{10}, t_{25} are 5th, 10th and 25th terms of an A.P. respectively, then the value of $\begin{vmatrix} t_5 & t_{10} & t_{25} \\ 5 & 10 & 25 \\ 1 & 1 & 1 \end{vmatrix}$ is equal to
(a) –40 (b) 1 (c) –1 (d) 0

9. Five dice are tossed. What is the probability that the five numbers shown will be different?
(a) $\frac{5}{24}$ (b) $\frac{5}{18}$ (c) $\frac{5}{27}$ (d) $\frac{5}{81}$

10. If the events A and B are independent and if $P(A^c) = \frac{2}{3}$, $P(B^c) = \frac{2}{7}$ then $P(A \cap B)$ is equal to
(a) $\frac{4}{21}$ (b) $\frac{3}{21}$ (c) $\frac{5}{21}$ (d) $\frac{1}{21}$

11. Let $P = [a_{ij}]$ be a 3×3 matrix and let $Q = [b_{ij}]$, where $b_{ij} = 2^{i+j} a_{ij}$ for $1 \le i, j \le 3$. If the determinant of P is 2, then the determinant of the matrix Q is
(a) 2^{10} (b) 2^{11} (c) 2^{12} (d) 2^{13}

12. $x(x^{n-1} - n\alpha^{n-1}) + \alpha^n(n-1)$ is divisible by $(x-\alpha)^2$ for
(a) $n > 1$ (b) $n > 2$
(c) $\forall\, n \in N$ (d) none of these

13. The sum of the series $1 + \frac{3^2}{2!} + \frac{3^4}{4!} + \frac{3^6}{6!} + \ldots$ to ∞ is
(a) e^{-3} (b) e^3
(c) $\frac{1}{2}(e^3 - e^{-3})$ (d) $\frac{1}{2}(e^3 + e^{-3})$

14. Value of the series $\frac{x}{1.2} + \frac{x^2}{2.3} + \frac{x^3}{3.4} + \ldots$ is
(a) $1 - \left(\frac{1-x}{x}\right)\log(1-x)$
(b) $1 - \left(\frac{1+x}{x}\right)\log(1+x)$
(c) $1 + \left(\frac{1-x}{x}\right)\log(1-x)$
(d) none of these

15. Let the co-efficients of powers of x in the 2nd, 3rd and 4th terms in the expansion of $(1+x)^n$, where n is a positive integer, be in arithmetic progression. The sum of the co-efficients of odd powers of x in the expansion is
(a) 32 (b) 64 (c) 128 (d) 256

16. The sum of the infinite series $1 + \frac{1}{3} + \frac{1\cdot3}{3\cdot6} + \frac{1\cdot3\cdot5}{3\cdot6\cdot9} + \frac{1\cdot3\cdot5\cdot7}{3\cdot6\cdot9\cdot12} + \ldots..$ is equal to
(a) $\sqrt{2}$ (b) $\sqrt{3}$ (c) $\sqrt{\frac{3}{2}}$ (d) $\sqrt{\frac{1}{\sqrt{3}}}$

17. The number of real values of α for which the system of equations
$x + 3y + 5z = \alpha x$
$5x + y + 3z = \alpha y$
$3x + 5y + z = \alpha z$
has infinite number of solution is
(a) 1 (b) 2 (c) 4 (d) 6.

18. Let S_K be the sum of an infinite G.P. series whose first term is K and common ratio is $\frac{K}{K+1}$ $(K > 0)$. Then the value of $\sum_{K=1}^{\infty} \frac{(-1)^K}{S_K}$ is equal to
(a) $\log_e 4$ (b) $\log_e 2 - 1$
(c) $1 - \log_e 2$ (d) $1 - \log_e 4$

19. Let A and B be two events with $P(A^c) = 0.3$, $P(B) = 0.4$ and $P(A \cap B^c) = 0.5$ then $P(B/A \cup B^c)$ is equal to
(a) $\frac{1}{4}$ (b) $\frac{1}{3}$ (c) $\frac{1}{2}$ (d) $\frac{2}{3}$

20. Three numbers are chosen at random without replacement from {1, 2, 3, …, 8}. The probability that their minimum is 3, given that their maximum is 6, is
(a) $\frac{1}{4}$ (b) $\frac{2}{5}$ (c) $\frac{3}{8}$ (d) $\frac{1}{5}$

21. If $C_0, C_1, C_2, C_3, \ldots$ are binomial co-efficients in the expansion of $(1+x)^n$. Then $\frac{C_0}{3} - \frac{C_1}{4} + \frac{C_2}{5} - \frac{C_3}{6} + \ldots$ is equal to
(a) $\frac{1}{n+1} - \frac{2}{n+2} + \frac{1}{n+3}$ (b) $\frac{1}{n+1} + \frac{2}{n+2} - \frac{1}{n+3}$
(c) $\frac{1}{n+1} - \frac{1}{n+2} + \frac{1}{n+3}$ (d) $\frac{2}{n+1} - \frac{1}{n+2} + \frac{2}{n+3}$

22. If $A = \begin{bmatrix} a & x \\ y & a \end{bmatrix}$ and if $xy = 1$. Then $det(AA^T)$ is equal to
(a) $a^2 - 1$ (b) $(a^2+1)^2$
(c) $1 - a^2$ (d) $(a^2-1)^2$

23. Let A and B any two events. Which one of the following statements is always true?
(a) $P(A^c/B) = P(A/B)$
(b) $P(A^c/B) = P(B^c/A)$
(c) $P(A^c/B) = 1 - P(A/B)$
(d) $P(A^c/B) = 1 - P(A/B^c)$

24. The inverse of a symmetric matrix is
(a) skew symmetric (b) symmetric
(c) diagonal matrix (d) none of thse

SOLUTIONS

1. (a): Given $P=\begin{bmatrix}1 & \alpha & 3\\ 1 & 3 & 3\\ 2 & 4 & 4\end{bmatrix}$

$|P| = 1(12-12)-\alpha(4-6)+3(4-6)$
$= 2\alpha - 6$

We know that $det(adjA) = (det\ A)^{n-1}$

$\therefore \quad 2\alpha - 6 = 4^{3-1} = 16$

$\Rightarrow \quad 2\alpha = 22$

$\therefore \quad \alpha = 11$

2. (b):
$$\left(\frac{x+1}{x^{\frac{2}{3}}-x^{\frac{1}{3}}+1}-\frac{x-1}{x-x^{\frac{1}{2}}}\right)^{10}$$
$$=\left\{\frac{\left(x^{\frac{1}{3}}+1\right)\left(x^{\frac{2}{3}}-x^{\frac{1}{3}}+1\right)}{x^{\frac{2}{3}}-x^{\frac{1}{3}}+1}-\frac{\left(\sqrt{x}+1\right)\left(\sqrt{x}-1\right)}{\sqrt{x}\left(\sqrt{x}-1\right)}\right\}^{10}$$
$$=\left(x^{\frac{1}{3}}+1-\frac{\sqrt{x}+1}{\sqrt{x}}\right)^{10}$$
$$=\left(x^{\frac{1}{3}}+1-1-x^{-\frac{1}{2}}\right)^{10}$$
$$=\left(x^{\frac{1}{3}}-x^{-\frac{1}{2}}\right)^{10}$$

Let t_{r+1} is independent of x.

$$\therefore \quad t_{r+1} = {}^{10}C_r\left(x^{\frac{1}{3}}\right)^{10-r}\left(-x^{-\frac{1}{2}}\right)^r$$
$$= (-1)^r\,{}^{10}C_r\,x^{\frac{10-r}{3}}\cdot x^{-\frac{r}{2}}$$
$$= (-1)^r\,{}^{10}C_r\,x^{\frac{20-5r}{6}}$$

Thus $\dfrac{20-5r}{6} = 0 \Rightarrow r = 4$

$\therefore$ The independent term is 5^{th} and its value is $(-1)^4\,{}^{10}C_4 = 210$

3. (b): $A = x(x^2-1) - 1(x-1) + 1(1-x)$
$= x^3 - x - x + 1 + 1 - x$
$A = x^3 - 3x + 2$

$$\Rightarrow \quad \frac{dA}{dx} = 3x^2 - 3 = 3(x^2-1)$$

$$\therefore \quad \frac{dA}{dx} = 3B \qquad \left[\because\ B=\begin{vmatrix}x & 1\\ 1 & x\end{vmatrix} = x^2-1\right]$$

4. (a): $C_0 + 2C_1 + 3C_2 + \ldots + (n+1)C_n$
$= (C_0 + C_1 + C_2 + \ldots + C_n) + (C_1 + 2C_2 + 3C_3 + \ldots + nC_n)$
$= 2^n + \left[n + 2\cdot\dfrac{n(n-1)}{2!} + 3\cdot\dfrac{n(n-1)(n-2)}{3!} + \ldots + n\right]$
$= 2^n + n\left[1 + (n-1) + \dfrac{(n-1)(n-2)}{2!} + \ldots + 1\right]$
$= 2^n + n(1+1)^{n-1}$
$= 2^n + n\cdot 2^{n-1}$
$= 2^n + n\cdot\dfrac{2^n}{2}$
$= (n+2)2^{n-1}$

$\therefore \quad (n+2)2^{n-1} = 576 = 9\times 2^6 = (7+2)2^{7-1}$

$\therefore \quad n = 7$

5. (c): $10^{10}\cdot(10^{10}+1)(10^{10}+2)$ is a product of 3 consecutive integers and hence is divisible by $3! = 6$

$\therefore$ remainder $= 0$

6. (b): Given that

$(1 + x + x^2)^n = 1 + a_1x + a_2x^2 + \ldots + a_{2n}x^{2n}$ …(i)

Putting $x = -1$, we get

$1 = 1 - a_1 + a_2 - a_3 + \ldots + a_{2n}$ …(ii)

Differentiating (i) w.r.t. x, we get

$n(1 + x + x^2)^{n-1}(1 + 2x) = a_1 + 2a_2x + \ldots + 2na_{2n}x^{2n-1}$

Putting $x = -1$, we get

$-n = a_1 - 2a_2 + 3a_3 - \ldots - 2na_{2n}$ …(iii)

(iii) – (ii) gives

$-n - 1 = -1 + 2a_1 - 3a_2 + 4a_3 - \ldots - (2n+1)a_{2n}$

$\Rightarrow \quad -n = 2a_1 - 3a_2 + 4a_3 - \ldots - (2n+1)a_{2n}$

7. (d): We have $\begin{vmatrix}\cos 2x & \sin 2x & \sin 2x\\ \sin 2x & \cos 2x & \sin 2x\\ \sin 2x & \sin 2x & \cos 2x\end{vmatrix} = 0$

$C'_1 = C_1 + C_2 + C_3$

$$\begin{vmatrix}\cos 2x + 2\sin 2x & \sin 2x & \sin 2x\\ \cos 2x + 2\sin 2x & \cos 2x & \sin 2x\\ \cos 2x + 2\sin 2x & \sin 2x & \cos 2x\end{vmatrix} = 0$$

$$\Rightarrow \quad (\cos 2x + 2\sin 2x)\begin{vmatrix}1 & \sin 2x & \sin 2x\\ 1 & \cos 2x & \sin 2x\\ 1 & \sin 2x & \cos 2x\end{vmatrix} = 0$$

$R'_1 = R_1 - R_2$ and $R'_2 = R_2 - R_3$

$$\Rightarrow \quad (\cos 2x + 2\sin 2x)\begin{vmatrix}0 & \sin 2x - \cos 2x & 0\\ 0 & \cos x - \sin 2x & \sin 2x - \cos 2x\\ 1 & \sin 2x & \cos 2x\end{vmatrix} = 0$$

$\Rightarrow \quad (\cos 2x + 2\sin 2x)(\sin 2x - \cos 2x)^2 = 0$

$\Rightarrow \quad \tan 2x = -\dfrac{1}{2}$ (rejected)

or $\quad \tan 2x = 1 = \tan\dfrac{\pi}{4}$

$\therefore \quad x = \dfrac{\pi}{8}$

8. (d): $t_5 = a + 4d$, $t_{10} = a + 9d$ and $t_{25} = a + 24d$

where a and d are respectively the 1st term and common difference.

$$\therefore \begin{vmatrix} t_5 & t_{10} & t_{25} \\ 5 & 10 & 25 \\ 1 & 1 & 1 \end{vmatrix} = \begin{vmatrix} a+4d & a+9d & a+24d \\ 5 & 10 & 25 \\ 1 & 1 & 1 \end{vmatrix}$$

Now $C'_1 = C_1 - C_2$ and $C'_2 = C_2 - C_3$ given

$$\begin{vmatrix} -5d & -15d & a+24d \\ -5 & -15 & 25 \\ 0 & 0 & 1 \end{vmatrix}$$

$= 1(75d - 75d)$

$= 0$

9. (a): Total number of cases $= 6^5$

No. of favourable cases $= 6! = 720$

Required probability $= \dfrac{720}{6^5} = \dfrac{5}{54}$

10. (c): Given that $P(A^c) = \dfrac{2}{3}$

$$\therefore \quad P(A) = \frac{1}{3}$$

and $P(B^c) = \dfrac{2}{7}$

$$\therefore \quad P(B) = \frac{5}{7}$$

Now $P(A \cap B) = P(A) \cdot P(B)$ [$\because$ A and B are independent]

$$= \frac{1}{3} \times \frac{5}{7}$$

$$P(A \cap B) = \frac{5}{21}$$

11. (d): Let $Q = \begin{vmatrix} 2^2a_{11} & 2^3a_{12} & 2^4a_{12} \\ 2^3a_{21} & 2^4a_{22} & 2^5a_{23} \\ 2^4a_{31} & 2^5a_{32} & 2^6a_{33} \end{vmatrix}$

$$= 2^2 \cdot 2^3 \cdot 2^4 \begin{vmatrix} a_{11} & 2a_{12} & 2^2a_{13} \\ a_{21} & 2a_{22} & 2^2a_{23} \\ a_{31} & 2a_{32} & 2^2a_{33} \end{vmatrix}$$

$$= 2^9 \cdot 2 \cdot 2^2 \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

$= 2^{12}\, det\, P$

$= 2^{12} \times 2$

$= 2^{13}$

12. (c): Let $S(n)x(x^{n-1} - n\alpha^{n-1}) + \alpha^n(n-1) = (x-\alpha)^2 g(x)$

$S(1) \equiv 0$ is true.

Let $S(m)$ is true

$\therefore \quad x(x^{m-1} - m\alpha^{m-1}) + \alpha^m(m-1) = (x-\alpha)^2 g(x)$

Now $S(m+1) \equiv x[x^m - (m+1)\alpha^m] + \alpha^{m+1}(m)$

$\equiv (x-\alpha)^2[xg(\lambda) + m\alpha^{m-1}]$ is true

So, holds $\forall\, n \in N$.

13. (d): $1 + \dfrac{3^2}{2!} + \dfrac{3^4}{4!} + \dfrac{3^6}{6!} + ...$

$$= \frac{1}{2}(e^3 + e^{-3}) \left[\because \quad e^x + e^{-x} = 2\left(1 + \frac{x^2}{2!} + \frac{x^4}{4!} + ...\infty\right)\right]$$

14. (c): $\dfrac{x}{1\cdot 2} + \dfrac{x^2}{2\cdot 3} + \dfrac{x^3}{3\cdot 4} + ...$

$$= \sum_{n=1}^{\infty} \frac{x^n}{n(n+1)}$$

$$= \sum_{n=1}^{\infty} \frac{\{(n+1) - n\}x^n}{n(n+1)}$$

$$= \sum_{n=1}^{\infty} \left(\frac{1}{n} - \frac{1}{n+1}\right)x^n$$

$$= \sum_{n=1}^{\infty} \frac{x^n}{n} - \frac{1}{x}\sum \frac{x^{n+1}}{n+1}$$

$$= -\log(1-x) + \frac{1}{x}[\log(1-x) + x]$$

$$= -\log(1-x) + \frac{1}{x}\log(1-x) + 1$$

$$= 1 + \left(\frac{1}{x} - 1\right)\log(1-x)$$

$$= 1 + \left(\frac{1-x}{x}\right)\log(1-x)$$

15. (b): Given that ${}^nC_1, {}^nC_2, {}^nC_3 \in$ A.P.

$\therefore \quad 2\,{}^nC_2 = {}^nC_1 + {}^nC_3$

$$\Rightarrow \quad 2 \cdot \frac{n(n-1)}{2!} = n + \frac{n(n-1)(n-2)}{3!}$$

$\Rightarrow \quad n^2 - 9n + 14 = 0$

$\Rightarrow \quad n = 7$ [$\because\, n \neq 2$]

$\therefore$ Sum of the co-efficients of odd powers of x in the expansion of $(1+x)^n$ is $2^{n-1} = 2^{7-1} = 2^6 = 64$

16. (b): $1 + \dfrac{1}{3} + \dfrac{1\cdot 3}{3\cdot 6} + \dfrac{1\cdot 3\cdot 5}{3\cdot 6\cdot 9} + \dfrac{1\cdot 3\cdot 5\cdot 7}{3\cdot 6\cdot 9\cdot 12} + ...$

$$= 1 + \frac{1}{3} + \frac{\frac{1}{2}\cdot\frac{3}{2}}{1\cdot 2}\left(\frac{2}{3}\right)^2 + \frac{\frac{1}{2}\cdot\frac{3}{2}\cdot\frac{5}{2}}{1\cdot 2\cdot 3}\left(\frac{2}{3}\right)^3 + ...$$

$$= 1 + \frac{1}{2}\cdot\frac{2}{3} + \frac{\frac{1}{2}\left(\frac{1}{2}+1\right)}{2!}\left(\frac{2}{3}\right)^2 + \frac{\frac{1}{2}\left(\frac{1}{2}+1\right)\left(\frac{1}{2}+2\right)}{3!}\left(\frac{2}{3}\right)^3 + ... \text{ to } \infty$$

$$= \left(1 - \frac{2}{3}\right)^{-\frac{1}{2}}$$

$$= 3^{\frac{1}{2}} = \sqrt{3}$$

17. (a): $(1-\alpha)x + 3y + 5z = 0$ ...(i)

$5x + (1-\alpha)y + 3z = 0$ ...(ii)

$$3x + 5y + (1-\alpha)z = 0 \quad \ldots(iii)$$

For infinite no. of solutions,

$$\begin{vmatrix} 1-\alpha & 3 & 5 \\ 5 & 1-\alpha & 3 \\ 3 & 5 & 1-\alpha \end{vmatrix} = 0$$

$C'_1 = C_1 + (C_2 + C_3)$

$$\Rightarrow \begin{vmatrix} 9-\alpha & 3 & 5 \\ 9-\alpha & 1-\alpha & 3 \\ 9-\alpha & 5 & 1-\alpha \end{vmatrix} = 0$$

$$\Rightarrow (9-\alpha)\begin{vmatrix} 1 & 3 & 5 \\ 1 & 1-\alpha & 3 \\ 1 & 5 & 1-\alpha \end{vmatrix} = 0$$

$$\Rightarrow (9-\alpha)\begin{vmatrix} 1 & 3 & 5 \\ 0 & -\alpha-2 & -2 \\ 0 & 2 & -\alpha-4 \end{vmatrix} = 0$$

$\Rightarrow (9-\alpha)[(\alpha+2)(\alpha+4)+4] = 0$

$\Rightarrow (9-\alpha)(\alpha^2+6\alpha+12) = 0$

$\Rightarrow \alpha = 9$ is the only real value of α.

18. (d): $S_1 = 1 + \frac{1}{2} + \frac{2}{2^2} + \ldots \text{to } \infty = \frac{1}{1-\frac{1}{2}} = 2$

$$S_2 = 2 + 2\cdot\frac{2}{3} + 2\left(\frac{2}{3}\right)^2 + \ldots \text{ to } \infty = \frac{2}{1-\frac{2}{3}} = 6$$

$$S_3 = 3 + 3\left(\frac{3}{4}\right) + 3\left(\frac{3}{4}\right)^2 + \ldots \text{ to } \infty = \frac{3}{1-\frac{3}{4}} = 12$$

$$S_4 = 4 + 4\left(\frac{4}{5}\right) + 4\cdot\left(\frac{4}{5}\right)^2 + \ldots \text{ to } \infty = \frac{4}{1-\frac{4}{5}} = 20$$

. . .
. . .
. . .
. . .

$$\therefore \sum_{K=1}^{\infty} \frac{(-1)^K}{S_K} = -\frac{1}{S_1} + \frac{1}{S_2} - \frac{1}{S_3} + \frac{1}{S_4} - \ldots \text{ to } \infty$$

$$= -\frac{1}{2} + \frac{1}{6} - \frac{1}{12} + \frac{1}{20} - \ldots \text{ to } \infty$$

$$= -\frac{1}{1\cdot 2} + \frac{1}{2\cdot 3} - \frac{1}{3\cdot 4} + \frac{1}{4\cdot 5} - \ldots \text{ to } \infty$$

$$= -\left(1-\frac{1}{2}\right) + \left(\frac{1}{2}-\frac{1}{3}\right) - \left(\frac{1}{3}-\frac{1}{4}\right) + \left(\frac{1}{4}-\frac{1}{5}\right) - \ldots \text{ to } \infty$$

$$= -1 + 2\left(\frac{1}{2} - \frac{1}{3} + \frac{1}{4} - \frac{1}{5} + \ldots \text{ to } \infty\right)$$

$$= -1 - 2\left(1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \ldots \text{ to } \infty\right) + 2$$

$$= -1 - 2\log_e 2 + 2 = 1 - \log_e 4$$

19. (a): $P(A/A\cup B^c) = \frac{P(B\cap(A\cup B^c))}{P(A\cup B^c)}$

$$= \frac{P[(B\cap A)\cup(B\cap B^c)]}{P(A)+P(B^c)-P(A\cap B^c)}$$

$$= \frac{P[(A\cap B)\cup\phi]}{(1-0.3)(1-0.4)-0.5}$$

$$= \frac{P(A\cap B)}{0.8}$$

$$= \frac{P(A)-P(A\cap B^c)}{0.8}$$

$$= \frac{(1-0.3)-0.5}{0.8}$$

$$= \frac{1}{4}$$

20. (d): Let A be the event that the maximum of chosen number is 6.

Let B be the event that the minimum of chosen number is 3.

$$\therefore \quad P(B/A) = \frac{P(A\cap B)}{P(B)} = \frac{\frac{1\cdot 1\cdot 2}{{}^8C_3}}{\frac{{}^5C_2}{{}^8C_3}} = \frac{2}{10} = \frac{1}{5}$$

21. (a)

22. (d): $A = \begin{bmatrix} a & x \\ y & a \end{bmatrix}$, $A^T = \begin{bmatrix} a & y \\ x & a \end{bmatrix}$

$$AA^T = \begin{bmatrix} a^2+x^2 & ay+xa \\ ya+ax & y^2+a^2 \end{bmatrix}$$

$$\det AA^T = (a^2+x^2)(y^2+a^2) - (ay+xa)^2$$
$$= a^2y^2 + x^2y^2 + a^4 + a^2x^2 - a^2y^2 - 2xya^2 - x^2a^2$$
$$= x^2y^2 + a^4 - 2xya^2$$
$$= (xy)^2 - 2a^2 + (a^2)^2$$
$$= (1)^2 - 2a^2 + (a^2)^2 \qquad [\because xy = 1]$$
$$= (1-a^2)^2$$
$$= (a^2-1)^2$$

23. (c): $P(A'/B) = \frac{P(A'\cap B)}{P(B)} = \frac{P(B)-P(A\cap B)}{P(B)}$

$$= 1 - \frac{P(A\cap B)}{P(B)}$$

$$= 1 - P(A/B)$$

24. (b): The inverse of a symmetric matrix is symmetric.

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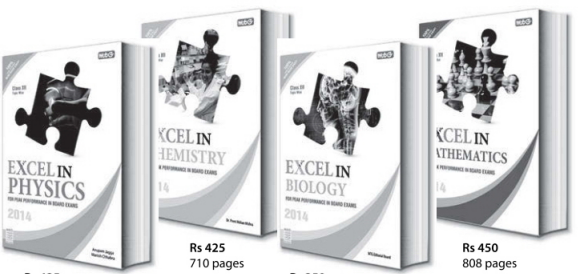

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STATISTICS

Definition : It is a study of the collection, organisation, analysis, interpretation and presentation of data. It deals with all aspects of data including the planning of data collection in terms of the design of surveys and experiments.

Measures of Central tendency : It gives us a rough idea where data points are centred. Mean, median and mode are three measures of central tendency.

Measures of Dispersion : The measures of central tendency are not sufficient to give complete information about a given data. So, the dispersion is the measure of variations in the values of the variable. It measures the degree of scatteredness of the observations in a distribution around the central value.

The various measures of dispersion are

(a) Range (b) Quartile Deviation

(c) Mean Deviation

(d) Standard Deviation

Range :

It is the difference between two extreme distributions *i.e.*, Range = Maximum Value – Minimum Value.

Mean Deviation : The mean deviation from '*a*' is denoted as M.D. (*a*)

$$\therefore \text{M.D.}(a) = \frac{\text{Sum of absolute values of deviations from } 'a'}{\text{Number of observations}}$$

(i) Mean deviation for ungrouped data

For n observations $x_1, x_2, \ldots, x_n$, the mean deviation about their mean $\bar{x}$ is given by

$$\text{M.D.}(\bar{x}) = \frac{1}{n}\sum_{n=1}^{n} |x_i - \bar{x}|$$

Mean deviation about their median is given by

$$\text{M.D.}(M)\ \frac{1}{n}\sum_{n=1}^{n} |x_i - M|$$

(ii) Mean deviation for discrete frequency distribution

Let the given data consist of discrete observations $x_1, x_2, \ldots, x_n$ occurring with the frequencies $f_1, f_2, \ldots, f_n$, respectively.

1. Mean deviation about mean

$$\text{M.D.}(\bar{x}) = \frac{1}{N}\sum_{i=1}^{n} f_i |x_i - \bar{x}|$$

where $\bar{x}$ = Mean, $N = \sum_{i=1}^{n} f_i$ = sum of frequencies

2. Mean deviation about median

$$\text{M.D.}(M) = \frac{1}{N}\sum_{i=1}^{n} f_i |x_i - M|$$

where M = Median

Mean deviation for continuous frequency distribution.

$$M.D.(\bar{x}) = \frac{\sum_{i=1}^{n} f_i |x_i - \bar{x}|}{N}$$

$$M.D.(M) = \frac{\sum_{i=1}^{n} f_i |x_i - M|}{N}$$

where x_i are midpoints of the classes, $\bar{x}$ and M are respectively, the mean and median of the distribution.

Shortcut method for calculating mean deviation about mean.

- Mean, $\bar{x} = a + \frac{\sum_{i=1}^{n} f_i d_i \times h}{N}$, where $d_i = \frac{x_i - a}{h}$

$$M.D.(\bar{x}) = \frac{1}{N}\sum_{i=1}^{n} f_i |x_i - \bar{x}|$$

- Median $= \frac{l + \frac{N}{2} - c.f}{f} \times h$

where median class is the class interval whose cumulative frequency is just greater than or equal to $\frac{N}{2}$, N is sum of frequencies, l, f, h and *c.f.* are respectively the lower limit, the frequency, the width of the median class and *c.f.* the cumulative frequency of the class just preceding the median class.

$$M.D.(M) = \frac{1}{N}\sum_{i=1}^{n} f_i |x_i - M|$$

Variance : The mean of the squares of the deviations from mean is called the variance and is denoted by σ^2

Variance for ungrouped data :

i.e. $\sigma^2 = \frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^2$

Variance for grouped data :

$$\sigma^2 = \frac{1}{n}\sum_{i=1}^{n} f_i(x_i - \bar{x})^2$$

Standard Deviation : The proper measure of dispersion about the mean of a set of observations is expressed as positive square root of the variance and is called standard deviation and is denoted by σ

Standard deviation for ungrouped data

$$\sigma = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^2}$$

Standard deviation for a discrete frequency distribution

$$\sigma^2 = \sqrt{\frac{1}{N}\sum_{i=1}^{n} f_i(x_i - \bar{x})^2}$$

where $N = \sum_{i=1}^{n} f_i$ and $\bar{x}$ = Mean

Standard deviation for continuous frequency distribution (grouped data) is

$$\sigma = \frac{1}{N}\sqrt{N\sum_{i=1}^{n} f_i x_i^2 - \left(\sum_{i=1}^{n} f_i x_i\right)^2}$$

where x_i are the midpoints of the classes and f_i their respective frequencies

Another Method for standard deviation

$$\sigma = \frac{h}{N}\sqrt{N\sum_{i=1}^{n} f_i y_1^2 - \left(\sum_{i=1}^{n} f_i y_i\right)^2}$$

where $y_i = \frac{x_i - A}{h}$

A = assumed mean

h = width of class intervals

Co-efficient of variation : The measure of variability which is independent of units is called co-efficient of variation and usually denoted by C.V.

$$\text{C.V.} = \frac{\sigma}{\bar{x}} \times 100, \bar{x} \neq 0$$

where σ = Standard deviation and $\bar{x}$ = Mean of the data

Comparing the variability or dispersion of two series, we calculate the coefficient of variance for each series. The series having greater C.V. is said to be more variable than the other.

The series having lesser C.V. is said to be more consistent than the other.

PROBABILITY

Definition : A measure of uncertainty or estimation of occurrence of an event. Probabilities are given a value between 0 (0% chance or will not happen) and 1 (100% chance or will happen).

Random Experiment : An experiment is said to be random experiment if it has more than one possible outcomes and it is not possible to predict the outcome in advance.

Outcome : A possible result of a random experiment is called its outcome.

Sample Space : The set of all possible outcomes of an experiment is called sample space.

Sample Points : Elements of sample space are called sample points.

Event : A subset of a sample space is called an event.

Occurrence of an Event : An event associated to a random experiment is said to occur if any one of the elementary events associated to it is an outcome.

Types of events

Impossible Event : The empty set 'ϕ' is called impossible events.

Sure Events : The whole sample space is called sure event.

Simple or Elementary Event : If an event E has only one sample point of a sample space it is called a simple (or elementary) event.

Algebra of Events

Complementary Event : For every event A, complement of A is the event consisting of all sample space outcomes that do not correspond to the occurrence of A. It is also called the event 'not A' or $S - A$.

Event 'A' or 'B' : It is the union of the events A & B *i.e.* $A \cup B = \{\omega : \omega \in A \text{ or } \omega \in B\}$

Event 'A' and 'B' : It is the intersection of events A & B *i.e.* $A \cap B = \{\omega : \omega \in A \text{ and } \omega \in B\}$

Event 'A' but not 'B' : An event $A - B$ in which elements are in A but not in B. Also $A - B = A \cap B'$

Mutually Exclusive Events : Two events A and B of a sample space S are mutually exclusive if the occurrence of any one of them excludes the occurrence of the other.

Hence, $A \cap B = \phi$, *i.e.*; A and B are disjoint sets.

Exhaustive Events : If $E_1, E_2, E_3, \ldots, E_n$ are n events of sample space S and if $E_1 \cup E_2 \cup \ldots \ldots \cup E_n = \bigcup_{i=1}^{n} E_i = S$. If $E_i \cap E_j = \phi$ for $i \neq j$, events E_i and E_j are pairwise disjoint and $\bigcup_{i=1}^{n} E_i = S$, then events $E_1, E_2, \ldots, E_n$ are called mutually exclusive and exhaustive events.

Equally like Outcomes : All outcomes with equal probability.

Classical definition : If all of the outcomes of a sample space are equally likely, then the probability that an event will occur is equal to the ratio:

$$\frac{\text{The number of outcomes favourable to the event}}{\text{The total number of outcomes of the sample space}}$$

Axiomatic approach to probability : Let S be a sample space containing elementary outcomes $w_1, w_2, \ldots, w_n$,

i.e., $S = \{w_1, w_2, \ldots, w_n\}$

It follows from the axiomatic definition of probability that

(i) $0 \leq P(w_i) \leq 1$ for each $w_i \in S$

(ii) $P(w_1) + P(w_2) + \ldots + P(w_n) = 1$

(iii) $P(A) = \Sigma P(w_i)$ for any event A containing elementary events w_i.

Probability of equally likely outcomes : Let a sample space of an experiment be $S = \{w_1, w_2, \ldots, w_n\}$ and suppose that all the outcomes are equally likely to occur *i.e.*, the chance of occurrence of each simple event must be the same

i.e., $P(w_i) = p$ for all $w_i \in S$, where $0 \leq p \leq 1$

Addition rule of probability : If A and B are any two events in a sample space S, then the probability that atleast one of the events A or B will occur is given by

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

Similarly, for three events A, B and C, we have

$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)$

Addition rule for mutually exclusive events : If A and B are disjoint sets, then

$P(A \cup B) = P(A) + P(B)$ [Since $P(A \cap B) = P(\phi) = 0$, where A and B are disjoint].

VERY SHORT ANSWER TYPE

1. There are 3 red and 3 black balls in a box. A die is rolled and if it shows an even number, a ball is drawn from the box. Write the sample space of the experiment.
2. Two dice are thrown simultaneously. Find the probability of getting a total of 10.
3. If three numbers are chosen from 1 to 20, then find the probability that they are not consecutive.
4. Find the probability that a leap year, selected at random, will contain 53 saturdays.
5. A card is drawn from a well shuffled deck of 52 cards. Find the probability of getting card drawn is either red or a king.
6. If A and B be the events in a sample space S such that $P(A) = \frac{3}{4}$, $P(B) = \frac{5}{8}$ and $P(A \cap B) = \frac{1}{2}$. Find $P(\bar{A} \cup \bar{B})$ and $P(\bar{A} \cap \bar{B})$

SHORT ANSWER TYPE

7. Find the mean, variance and the standard deviation for the following data:
   5, 9, 8, 12, 6, 10, 6, 8
8. A box contains 20 red marbles, 10 blue marbles and 30 green marbles. 5 marbles are drawn from the box, what is the probability that (i) all will be red? (ii) atleast one will be blue?
9. From a well shuffled deck of 52 cards. 4 cards are drawn then find the probability of getting one spade and three hearts.
10. Three squares of normal chessboard are chosen. Find the probability of getting 2 squares of one colour and the other of different colour.
11. The letter of the word SOCIETY are placed at random in a row. Find the probability that the three vowels did not come together.
12. Find the mean deviation about the mean for the following data:
    6, 8, 10, 12, 13, 9, 8, 20, 4
13. Find the mean deviation about the median for the following data:
    320, 150, 210, 280, 300, 310, 340
14. Find mean and variance of 6, 8, 10, 12, 14
15. Some particulars of the distribution of weights of boys and girls in a class are given below:

	Boys	Girls
Number	100	50
Mean Weight	60 kg	45 kg
Variance	25	9

Which of the distributions is more variable?

LONG ANSWER TYPE

16. Two cards are drawn at random from a well-shuffled pack of 52 cards. What is the probability that either both are red or both are kings?

17. A box contains 100 bolts and 50 nuts. It is given that 50% bolts and 50% nuts are rusted. Two objects are selected from the box at random. Find the probability that either both are bolts or both are rusted.

18. The mean and variance of seven observations are 8 and 16 respectively. If five of these are 2, 4, 10, 12, 14; find the remaining two observations.

SOLUTIONS

1. When a die is rolled then $S_1 = \{1, 2, 3, 4, 5, 6\}$
When a ball is drawn then $S_2 = \{R_1, R_2, R_3, B_1, B_2, B_3\}$
Hence, Sample space = $\{1, 3, 5, 2R_1, 4R_1, 6R_1, 2R_2, 4R_2, 6R_2, 2R_3, 4R_3, 6R_3, 2B_1, 4B_1, 6B_1, 2B_2, 4B_2, 6B_2, 2B_3, 4B_3, 6B_3\}$.

2. When two dice are thrown total number of outcomes = $6^2 = 36$
Let event A be getting total of 10
then $A = \{(4, 6), (5, 5), (6, 4)\}$
Thus favourable outcomes = 3

$\therefore$ Reqd. Probability $= \frac{3}{36} = \frac{1}{12}$

3. As there are 18 types of consecutive numbers from 1 to 20

$\therefore$ Required Probability $= 1 - \frac{18}{{}^{20}C_3}$

$= 1 - \frac{18}{1140} = \frac{1122}{1140} = \frac{187}{190}$

4. In a leap year, there are 366 days *i.e.*, 52 weeks and 2 days
Thus, a leap year has always 53 saturdays
The remaining two days can be :
(i) Sun. & Mon. (ii) Mon. & Tues.
(iii) Tues. & Wed. (iv) Wed. & Thurs.
(v) Thurs. & Fri. (vi) Fri. & Sat.
(vii) Sat. & Mon.
$\therefore$ Favourable outcomes = 2 *i.e.*
(i) Friday & Saturday (ii) Saturday & Monday.

Thus, required probability $= \frac{2}{7}$

5. A pack of 52 cards has 26 red cards which include 2 red kings and 2 black king.
$\therefore$ Number of cards which are red or king cards = 26 + 2 = 28

Required Probability $= \frac{28}{52} = \frac{7}{13}$

6. Here $P(A) = \frac{3}{4}, P(B) = \frac{5}{8}$ and $P(A \cap B) = \frac{1}{2}$

$\therefore \quad P(\bar{A} \cup \bar{B}) = P(\overline{A \cup B}) = 1 - P(A \cap B)$

$= 1 - \frac{1}{2} = \frac{1}{2}.$

Also, $P(\bar{A} \cap \bar{B}) = P(\overline{A \cap B}) = 1 - P(A \cup B)$

We know that $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= \frac{3}{4} + \frac{5}{8} - \frac{1}{2} = \frac{7}{8}$

$\therefore \quad P(\bar{A} \cap \bar{B}) = 1 - \frac{7}{8} = \frac{1}{8}$

7. Here $n = 8$.

Mean, $\bar{x} = \frac{1}{8}(5 + 9 + 8 + 12 + 6 + 10 + 6 + 8) = \frac{64}{8} = 8.$

The values of $(x_i - \bar{x})$ are
$-3, 1, 0, 4, -2, 2, -2, 0.$

$\therefore \quad \Sigma(x_i - \bar{x})^2 = (9 + 1 + 0 + 16 + 4 + 4 + 4 + 0) = 38.$

$\therefore$ variance, $\sigma^2 = \frac{\Sigma(x_i - \bar{x})^2}{n} = \frac{38}{8} = \frac{19}{4} = 4.75.$

and, standard deviation, $\sigma = \sqrt{4.75} = 2.17$

$\therefore$ mean = 8, variance = 4.75 and standard deviation = 2.17.

8. Here total no. of marbles = 20 + 10 + 30 = 60.
5 marbles are drawn out of 60 marbles in ${}^{60}C_5$ ways.
(i) There are 20 red marbles.
We have to draw 5 marbles out of 20 red marbles in ${}^{20}C_5$ ways.

$\therefore$ Probability that all will be red $= \frac{{}^{20}C_5}{{}^{60}C_5}$

(ii) There are 20 red marbles and 30 green marbles. We have to draw 5 marbles out of 50 marbles in ${}^{50}C_5$ ways.
Required Probability $= 1 - \frac{{}^{50}C_5}{{}^{60}C_5}$

9. 4 cards are drawn from a well-shuffled deck of 52 cards in ${}^{52}C_4$ ways.
There are 13 cards of spades and 13 cards of hearts.
Now 1 card of spade out of 13 cards can be drawn in ${}^{13}C_1$ ways and 3 cards of hearts out of 13 cards of hearts can be drawn in ways ${}^{13}C_3$
Probability of obtaining one spade and three hearts $= \frac{{}^{13}C_1 \times {}^{13}C_3}{{}^{52}C_4}$

10. A chessboard has 64 squares. Out of which 32 are white and 32 are black. As three squares are chosen they can be 2 whites and 1 black or 1 white and two blacks.

$$= \frac{{}^{32}C_2 \times {}^{32}C_1 + {}^{32}C_2 \times {}^{32}C_1}{{}^{64}C_3}$$

$$= 2\frac{{}^{32}C_2 \times {}^{32}C_1}{{}^{64}C_3} = \frac{16}{21}.$$

11. The word 'SOCIETY' has 7 different letters and they can be arranged in 7! *i.e.* 5040 ways. We can arrange four letters S, C, T, Y and a pack of three vowels in 5! = 120 ways.

Also, three vowels in their pack can be arranged in 3! *i.e.* 6 ways.

Hence, the number of arrangement in which 3 vowels come together = 120 × 6 = 720.

$$P(\text{three vowels come together}) = \frac{720}{5040} = \frac{1}{7}$$

$\therefore$ P(three vowels do not come together)

$$= 1 - \frac{1}{7} = \frac{8}{7}$$

12. We have to first find the mean $(\bar{x})$ of the given data

$$\bar{x} = \frac{6+8+10+12+13+9+8+20+4}{9} = \frac{90}{9} = 10$$

x_i	6	8	10	12	13	9	8	20	4
$\lvert x_i - x \rvert$	4	2	0	2	3	1	2	10	6

$$\sum_{x=1}^{9} |x_i - \bar{x}| = 30$$

$$\text{M.D.}(\bar{x}) = \frac{\sum_{i=1}^{n} |x_i - \bar{x}|}{n} = \frac{30}{9} = 3.33$$

13. Arranging the observations in ascending order 150, 210, 280, 300, 310, 320, 340

$$\text{Median} = \left(\frac{n+1}{2}\right)^{\text{th}} \text{term}$$

$\therefore$ $$\text{Median} = \left(\frac{7+1}{2}\right)^{\text{th}} \text{term} = 4^{\text{th}} \text{term}$$

$\therefore$ Median (M) = 300

x_i	320	150	210	280	300	310	340
$\lvert x_i - 300 \rvert$	20	150	90	20	0	10	40

$\Sigma |x_i - 300| = 330$

$$\text{M.D. } (M) = \frac{1}{7} \times 330 = 47.14$$

14. Mean $\bar{x} = \dfrac{6+8+10+12+14}{5} = \dfrac{50}{5} = 10$

x_i	6	8	10	12	14
$(x_i - 10)$	–4	–2	0	2	4
$(x_i - 10)^2$	16	4	0	4	16

$\Sigma(x_i - 10)^2 = 40$

$$\text{Variance, } \sigma^2 = \frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^2 = \frac{1}{5} \times 40 = 8$$

15. Distribution with more C.V is more variable.

We know that, C.V. $= \dfrac{\sigma}{x} \times 100,\ \bar{x} \neq 0$

where σ = standard deviation

& $\bar{x}$ = Mean

$\therefore$ σ(for Boys) $= \sqrt{25} = 5$

& σ(for Girls) $= \sqrt{9} = 3$

Thus, C.V. (Boys) $= \dfrac{5}{6} \times 10 = 8.33$ (approx)

& C.V. (Girls) $= \dfrac{3}{45} \times 100 = 6.67$ (approx)

16. Let S be the sample space. Then,

$n(S)$ = number of ways of drawing 2 cards out of 52 $= {}^{52}C_2$.

Let E_1 = event that both are red cards,

and E_2 = event that both are kings.

Then, $(E_1 \cap E_2)$ = event of getting 2 red kings.

$\therefore$ $n(E_1)$ = number of ways of drawing 2 red cards out of 26 red cards = ${}^{26}C_2$.

$\therefore$ $n(E_1 \cap E_2)$ = number of ways of drawing 2 red kings out of 2 red kings = ${}^{2}C_2 = 1$.

$\therefore$ $$P(E_1) = \frac{n(E_1)}{n(S)} = \frac{{}^{26}C_2}{{}^{52}C_2};\ P(E_2) = \frac{n(E_2)}{n(S)} = \frac{{}^{4}C_2}{{}^{52}C_2}$$

and $$P(E_1 \cap E_2) = \frac{n(E_1 \cap E_2)}{n(S)} = \frac{1}{{}^{52}C_2}.$$

$\therefore$ P(drawing both red cards or both kings)

$= P(E_1 \text{ or } E_2) = P(E_1 \cup E_2)$

$= P(E_1) + P(E_2) - P(E_1 \cap E_2)$

$$= \left(\frac{{}^{26}C_2}{{}^{52}C_2} + \frac{{}^{4}C_2}{{}^{52}C_2} - \frac{1}{{}^{52}C_2} \right) = \frac{{}^{26}C_2 + {}^{4}C_2 - 1}{{}^{52}C_2}$$

$$= \frac{(325 + 6 - 1)}{1326} = \frac{330}{1326} = \frac{55}{221}$$

Hence, the required probability is $\frac{55}{221}$.

17. Total number of objects = (100 + 50) = 150.

Let S be the sample space. Then,

$n(S)$ = number of ways of selecting 2 objects out of 150 = ${}^{150}C_2$.

Number of rusted objects

= (50% of 100) + (50% of 50) = (50 + 25) = 75.

Let E_1 = event of selecting 2 bolts out of 100 bolts, and E_2 = event of selecting 2 rusted objects out of 75 rusted objects.

$\therefore$ $(E_1 \cap E_2)$ = event of selecting 2 rusted bolts out of 50 rusted bolts

$\therefore$ $n(E_1)$ = number of ways of selecting 2 bolts out of 100 = ${}^{100}C_2$.

$\therefore$ $n(E_2)$ = number of ways of selecting 2 rusted objects out of 75 = ${}^{75}C_2$.

$\therefore$ $n(E_1 \cap E_2)$ = number of ways of selecting 2 rusted bolts out of 50 = ${}^{50}C_2$.

$$\therefore \quad P(E_1) = \frac{n(E_1)}{n(S)} = \frac{{}^{100}C_2}{{}^{150}C_2}, \; P(E_2) = \frac{n(E_2)}{n(S)} = \frac{{}^{75}C_2}{{}^{150}C_2}$$

and $P(E_1 \cap E_2) = \dfrac{n(E_1 \cap E_2)}{n(S)} = \dfrac{{}^{50}C_2}{{}^{150}C_2}$

P(selecting both bolts or both rusted objects)

$= P(E_1 \text{ or } E_2) = P(E_1 \cup E_2)$

$= P(E_1) + P(E_2) - P(E_1 \cap E_2)$

$$= \frac{{}^{100}C_2}{{}^{150}C_2} + \frac{{}^{75}C_2}{{}^{150}C_2} - \frac{{}^{50}C_2}{{}^{150}C_2} = \frac{\left({}^{100}C_2 + {}^{75}C_2 - {}^{50}C_2\right)}{{}^{150}C_2}$$

$$= \frac{(4950 + 2775 - 1225)}{11175} = \frac{6500}{11175} = \frac{260}{447} = 0.58$$

Hence, the required probability is 0.58.

18. Let the remaining two observations be x by y. Then,

$$\text{mean} = 8 \Rightarrow \frac{2 + 4 + 10 + 12 + 14 + x + y}{7} = 8$$

$$\Rightarrow 42 + x + y = 56$$

$$\Rightarrow x + y = 14. \qquad \ldots(i)$$

Also, variance = 16

$$\Rightarrow \quad \frac{1}{7}(2^2 + 4^2 + 10^2 + 12^2 + 14^2 + x^2 + y^2) - 8^2 = 16$$

$$\left[\because \; \sigma^2 = \frac{\Sigma x_i^2}{n} - (\bar{x})^2 \right]$$

$$\Rightarrow \quad \frac{1}{7}(460 + x^2 + y^2) = 80$$

$$\Rightarrow \quad 460 + x^2 + y^2 = 560$$

$$\Rightarrow \quad x^2 + y^2 = 100. \qquad \ldots(ii)$$

Now, $(x + y)^2 + (x - y)^2 = 2(x^2 + y^2)$

$$\Rightarrow \quad (x - y)^2 = 2(x^2 + y^2) - (x - y)^2$$

$$\Rightarrow \quad (x - y)^2 = 2(x^2 + y^2) - (x + y)^2$$

$$\Rightarrow \quad (x - y)^2 = (2 \times 100) - (14)^2 = (200 - 196) = 4$$

$$\Rightarrow \quad x - y = \pm 2.$$

Now, $x + y = 14$, $x - y = 2 \Rightarrow x = 8$, $y = 6$;

$x + y = 14$, $x - y = -2 \Rightarrow x = 6$, $y = 8$.

Hence, the remaining two observations are 6 and 8.

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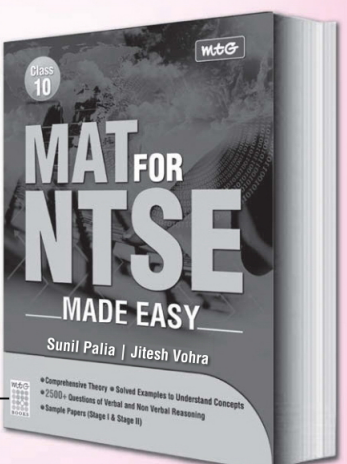

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SAMPLE PAPER

CBSE Board 2014

Time : 3 hrs. Marks : 100

GENERAL INSTRUCTIONS

(i) All questions are compulsory.
(ii) Question numbers 1 to 10 are very short answer questions and carry 1 mark each.
(iii) Question numbers 11 to 22 are short answer questions and carry 4 marks each.
(iv) Question numbers 23 to 29 are long answer type questions and carry 6 marks each.

SECTION - A

1. Using principal values, find the value of $\cos^{-1}\left(\frac{-1}{2}\right) + \tan^{-1}(1)$.

2. Evaluate $\tan\left(\sin^{-1}\left(\sin\frac{2\pi}{3}\right)\right)$

3. Write the value of $3xyz$, if $\begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 4 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ -1 \\ 8 \end{bmatrix}$

4. If A is square matrix of 3×3 such that $|\text{adj } A| = 324$ then find $|A|$.

5. Write the inverse of the matrix $A = \begin{bmatrix} -1 & 4 \\ 7 & -20 \end{bmatrix}$ using elementary row transformation.

6. The total revenue received from the sale of x units of a product is given by $R(x) = 15x^2 + 4x + 30$. Find the marginal revenue when $x = 5$, where by marginal revenue we mean the rate of $R(x)$ with respect to the number of items sold at an instant.

7. Determine the order and degree of differential equation $\frac{d^3x}{dt^3} + \frac{d^2x}{dt^2} + \left(\frac{dx}{dt}\right)^2 = e^t$.

8. If the position vector $\vec{a}$ of a point $(12, n)$ is such that $|\vec{a}| = 13$, find the value of n.

9. If $\vec{a} = 7\hat{i} + \hat{j} - 4\hat{k}$ and $\vec{b} = 2\hat{i} + 6\hat{j} + 3\hat{k}$, find the projection of $\vec{b}$ on $\vec{a}$.

10. Find the direction cosines of the line $\frac{x-2}{2} = \frac{2y-5}{-3} = z + 1$

SECTION - B

11. Let S and T be two sets. Show that $f : S \times T \to T \times S$ defined as $f(s, t) = (t, s)$ is one-one and onto.

12. Simplify $\cos^{-1}\left(\frac{3}{5}\cos x + \frac{4}{5}\sin x\right)$

OR

Prove that

$$\tan^{-1}\left\{\frac{\sqrt{1+x^2}+\sqrt{1-x^2}}{\sqrt{1+x^2}-\sqrt{1-x^2}}\right\} = \frac{\pi}{4} + \frac{1}{2}\cos^{-1}x^2; -1 < x < 1$$

13. Three shopkeepers A, B, C are using polythene, handmade bag and newspaper envelopes as carry bags. it is found that the shopkeepers A, B, C are using (20, 30, 40), (30, 40, 20), (40, 20, 30) polythene, handmade bags and newspapers envelopes respectively. The shopkeepers A, B, C spend ₹ 250, ₹ 270 & ₹ 200 on these carry bags respectively.

(i) Represent the above situation by a Matrix equation and form Linear equations using matrix multiplication.

(ii) Is it possible to find the cost of each carry bags using matrices.

(iii) Keeping in mind the social & environmental conditions, which shopkeeper is better? & why ?

14. If $y = \frac{\sqrt{1-x^2}(2x-3)^{1/2}}{(x^2+2)^{2/3}}$, find $\frac{dy}{dx}$.

15. Given that $\cos\frac{x}{2} \cdot \cos\frac{x}{4} \cdot \cos\frac{x}{8} \ldots = \frac{\sin x}{x}$, prove that

$$\frac{1}{2^2}\sec^2\frac{x}{2}+\frac{1}{2^4}\sec^2\frac{x}{4}+\ldots=\operatorname{cosec}^2 x-\frac{1}{x^2}$$

16. Let $f(x)=\begin{cases}\dfrac{1-\cos 4x}{x^2} & ,\text{ if } x<0\\ a & ,\text{ if } x=0\\ \dfrac{\sqrt{x}}{\sqrt{16+\sqrt{x}}-4} & ,\text{ if } x>0\end{cases}$

Determine the value of a so that $f(x)$ is continuous at $x = 0$.

OR

If $\log(x^2+y^2)=2\tan^{-1}\left(\frac{y}{x}\right)$, show that $\frac{dy}{dx}=\frac{x+y}{x-y}$

17. $\int\tan^{-1}(\sec x+\tan x)dx, -\pi/2<x<\pi/2$

OR

$$\int\left\{(2x-3)^5+\frac{1}{(7x-5)^3}+\frac{1}{\sqrt{5x-4}}+\frac{1}{2-3x}+\sqrt{3x+2}\right\}dx$$

18. Evaluate : $\int\frac{1}{\sqrt[4]{(x-1)^3(x+2)^5}}dx$

19. Prove that : $\int_0^{\pi}\frac{x}{1-\cos\alpha\sin x}dx=\frac{\pi(\pi-\alpha)}{\sin\alpha}$

20. If $\vec{a}$ and $\vec{b}$ are non-collinear vectors and vectors $\vec{\alpha}=(x+4y)\vec{a}+(2x+y+1)\vec{b}$ and $\vec{\beta}=(-2x+y+2)\vec{a}+(2x-3y-1)\vec{b}$ are connected by the relation $3\vec{\alpha}=2\vec{\beta}$, find x, y.

21. Show that the lines

$$\frac{x-a+d}{\alpha-\delta}=\frac{y-a}{\alpha}=\frac{z-a-d}{\alpha+\delta}$$

and $\frac{x-b+c}{\beta-\gamma}=\frac{y-b}{\beta}=\frac{z-b-c}{\beta+\gamma}$ are coplanar.

OR

Find the length and the foot of the perpendicular from the point (7, 14, 5) to the plane $2x+4y-z=2$.

22. A clever student used a biased coin so that the head is 3 times as likely to occur as tail. If the coin tossed twice find the probability distribution and mean of numbers of tails. Is this a good tendency? Justify your answer.

SECTION - C

23. If $A=\begin{bmatrix}1 & \tan x\\ -\tan x & 1\end{bmatrix}$, show that

$$A^TA^{-1}=\begin{bmatrix}\cos 2x & -\sin 2x\\ \sin 2x & \cos 2x\end{bmatrix}$$

24. Find the points on the curve $9y^2 = x^3$ where normal to the curve makes equal intercepts with the axes.

OR

Show that the triangle of maximum area that can be inscribed in a given circle is an equilateral triangle.

25. Find the area of the region $\{(C, y) : x^2+y^2\le 1\le x+y\}$.

26. Solve the following initial value problem.

$(xe^{y/x}+y)dx = xdy,\ y(1) = 1$

27. Show that the lines $\frac{x-1}{3}=\frac{y+1}{2}=\frac{z-1}{5}$ and $\frac{x+2}{4}=\frac{y-1}{3}=\frac{z+1}{-2}$ do not intersect.

28. A dietician wishes to mix two types of food in such a way that the vitamin content of the mixture contain at least 8 units of vitamin *A* and 10 units of vitamin *C*. Food I contains 2 units per kg of vitamin *A* and 1 unit per kg of vitamin *C*, while food II contain 1 unit per kg of vitamin *A* and 2 units per kg of vitamin *C*. It cost ₹ 5.00 per kg to purchase food I and ₹ 7.00 per kg to purchase food II. Determine the minimum cost of the mixture. Formulate the LPP and solve it. Why a person should take balanced food?

29. There are 20 people in group. Out of them 7 people are non-vegetarian, 2 people are selected randomly. Write the probability distribution of non-vegetarian people. Explain whether you would like to be vegetarian or non-vegetarian and why? Also keeping life of animals in mind, how would you promote a person to be vegetarian?

OR

A chairman is biased so that he selects his relatives for a job 3 times as likely as others. If there are 3 posts for a job, find the probability distribution for selection of persons other than their relatives. If the chairman is biased than which value of life will be demolished?

SOLUTIONS

1. $\cos^{-1}\left(\frac{-1}{2}\right)=$ An angle in $[0, \pi]$

$\Rightarrow \quad \cos^{-1}\left(\frac{-1}{2}\right)=\cos^{-1}\left(\frac{1}{2}\right)=\frac{2\pi}{3}$

Similarly, $\tan^{-1}(1)$ = An angle in $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

$\Rightarrow \quad \tan^{-1}(1) = \frac{\pi}{4}$

$\cos^{-1}\left(\frac{-1}{2}\right) + \tan^{-1}(1) = \frac{2\pi}{3} + \frac{\pi}{4}$

$= \frac{8\pi + 3\pi}{12} = \frac{11\pi}{12}$

2. $\sin^{-1}\left(\sin\frac{2\pi}{3}\right) \neq \frac{2\pi}{3}$ as $\frac{2\pi}{3}$ does not lie between $-\frac{\pi}{2}$ and $\frac{\pi}{2}$.

Now, $\sin^{-1}\left(\sin\frac{2\pi}{3}\right) = \sin^{-1}\left(\sin\left(\pi - \frac{\pi}{3}\right)\right)$

$= \sin^{-1}\left(\sin\frac{\pi}{3}\right) = \frac{\pi}{3}$

Hence, $\tan\frac{\pi}{3} = \sqrt{3}$

3. $\begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 4 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2x \\ 3y \\ 4z \end{bmatrix}$

$\Rightarrow \quad 2x = 4,\ 3y = -6,\ 4z = 8$

$\Rightarrow \quad x = 2,\ y = -2,\ z = 2$

$\Rightarrow \quad 3xyz = 3 \times 2 \times (-2) \times 2 = -24$

4. If A be a non-singular square matrix of order $n \times n$, then $|\text{adj } A| = |A|^{n-1}$

$|\text{adj } A| = 324 = |A|^{n-1} = |A|^2 = 324 \Rightarrow |A| = 18$

5. We have $A = IA$

$\begin{bmatrix} -1 & 4 \\ 7 & -20 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}A$

Applying $R_2 \to R_2 + 7R_1$, we get

$\begin{bmatrix} -1 & 4 \\ 0 & 8 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 7 & 1 \end{bmatrix}A$

$R_2 \to R_2/8$

$\begin{bmatrix} -1 & 4 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 7/8 & 1/8 \end{bmatrix}A$

Applying $R_1 \to R_1 - 4R_2$, we get

$\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} -5/2 & -1/2 \\ 7/8 & 1/8 \end{bmatrix}A$

$R_1 \leftrightarrow -R_1$

$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 5/2 & 1/2 \\ 7/8 & 1/8 \end{bmatrix}A$

$\Rightarrow \quad A^{-1} = \begin{bmatrix} 5/2 & 1/2 \\ 7/8 & 1/8 \end{bmatrix} = \frac{1}{8}\begin{bmatrix} 20 & 4 \\ 7 & 1 \end{bmatrix}$

6. Marginal revenue $= \frac{dR}{dx}$

$= \frac{d}{dx}(15x^2 + 4x + 30) = 15 \times 2x + 4$

when $x = 5$, we have

Marginal Revenue $= 15 \times 10 + 4 = 150 + 4 = 154$

Hence, the required marginal revenue is ₹ 154.

7. Order = 3, degree = 1

8. The position vector of the point (12, n) is $12\hat{i} + n\hat{j}$

$\therefore \quad \vec{a} = 12\hat{i} + n\hat{j} \Rightarrow |\vec{a}| = \sqrt{12^2 + n^2} = 13$

$\Rightarrow \quad 12^2 + n^2 = 169 \Rightarrow n^2 = 25 \Rightarrow n = \pm 5$

9. Projection of $\vec{b}$ on $\vec{a} = \frac{\vec{b} \cdot \vec{a}}{|\vec{a}|}$

$= \frac{14 + 6 - 12}{\sqrt{49 + 1 + 16}} = \frac{8}{\sqrt{66}}$

10. The given line is

$\frac{x-2}{2} = \frac{2y-5}{-3} = \frac{z+1}{0} \Rightarrow \frac{x-2}{2} = \frac{y-5/2}{-3/2} = \frac{z+1}{0}$

Direction cosines are

$\frac{2}{\sqrt{2^2 + \left(\frac{-3}{2}\right)^2 + 0^2}}, \frac{-3/2}{\sqrt{2^2 + \left(\frac{-3}{2}\right)^2 + 0^2}}, \frac{0}{\sqrt{2^2 + \left(\frac{-3}{2}\right)^2 + 0^2}}$

$\Rightarrow \quad \frac{2}{5/2}, \frac{-3/2}{5/2}, 0 \Rightarrow \frac{4}{5}, \frac{-3}{5}, 0$

11. **One-One :** Let (s_1, t_1) and $(s_2, t_2) \in S \times T$ such that

$f(s_1, t_1) = f(s_2, t_2)$

$\Rightarrow \quad (t_1, s_1) = (t_2, s_2) \Rightarrow t_1 = t_2$ and $s_1 = s_2$

$\Rightarrow \quad (s_1, t_1) = (s_2, t_2)$

Thus, $f(s_1, t_1) = f(s_2, t_2) \Rightarrow (s_1, t_1) = (s_2, t_2)$ for all $(s_1, t_1), (s_2, t_2) \in S \times T$.

So, f is an one-one

onto : Let (t, s) be an arbitrary element of $T \times S$. Then, $s \in T$ and $s \in S$

$\Rightarrow \quad (s, t) \in S \times T$.

Thus, for all $(t, s) \in T \times S$ there exists $(s, t) \in S \times T$ such that $f(s, t) = (t, s)$

So, $f : S \times T \to T \times S$ is an onto function.

Hence, f is one-one and onto

12. (i) Let $\frac{3}{5} = r\cos\theta$ and $\frac{4}{5} = r\sin\theta$

$\Rightarrow \quad r = \sqrt{\left(\frac{3}{5}\right)^2 + \left(\frac{4}{5}\right)^2} = 1$ and $\tan\theta = \frac{r\sin\theta}{r\cos\theta} = \frac{4}{3}$

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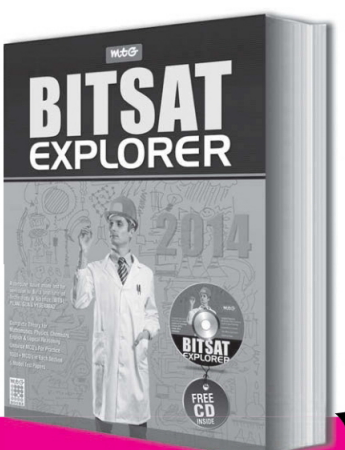

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$\Rightarrow \quad r = 1$ and $\theta = \tan^{-1}\frac{4}{3}$

$\Rightarrow \quad \frac{3}{5} = \cos\theta$ and $\frac{4}{5} = \sin\theta$, where $\theta = \tan^{-1}\frac{4}{3}$

$\Rightarrow \quad \cos^{-1}\left(\frac{3}{5}\cos x + \frac{4}{5}\sin x\right) = \cos^{-1}(\cos\theta\cos x + \sin\theta\sin x)$

$= \cos^{-1}\{\cos(x-\theta)\} = x - \theta = x - \tan^{-1}\frac{4}{3}.$

OR

Putting $x^2 = \cos 2\theta$, we have

$$\tan^{-1}\left\{\frac{\sqrt{1+\cos 2\theta}+\sqrt{1-\cos 2\theta}}{\sqrt{1+\cos 2\theta}-\sqrt{1-\cos 2\theta}}\right\}$$

$$= \tan^{-1}\left\{\frac{\sqrt{2\cos^2\theta}+\sqrt{2\sin^2\theta}}{\sqrt{2\cos^2\theta}-\sqrt{2\sin^2\theta}}\right\}$$

$$= \tan^{-1}\left\{\frac{\cos\theta+\sin\theta}{\cos\theta-\sin\theta}\right\} = \tan^{-1}\left\{\frac{1+\tan\theta}{1-\tan\theta}\right\}$$

$$= \tan^{-1}\left\{\tan\left(\frac{\pi}{4}+\theta\right)\right\}$$

$$= \frac{\pi}{4}+\theta \quad \left[\because \quad -1 < x < 1 \Rightarrow 0 < x^2 < 1 \Rightarrow 0 < 2\theta < \frac{\pi}{2} \right.$$
$$\left. \Rightarrow \quad 0 < \theta < \frac{\pi}{4}\right]$$

$$= \frac{\pi}{4} + \frac{1}{2}\cos^{-1}x^2 \qquad [\because x^2 = \cos 2\theta \Rightarrow 2\theta = \cos^{-1}x^2]$$

13. **(i)** Let x, y, z be the cost polythene, handmade bags, newspaper envelopes respectively used by shopkeepers A, B, C.

$$\begin{bmatrix} 20 & 30 & 40 \\ 30 & 40 & 20 \\ 40 & 20 & 30 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 250 \\ 270 \\ 200 \end{bmatrix}$$

$20x + 30y + 40z = 250$
$30x + 40y + 20z = 270$
$40x + 20y + 30z = 200$

(ii) let $P = 1000\begin{bmatrix} 2 & 3 & 4 \\ 3 & 4 & 2 \\ 4 & 2 & 3 \end{bmatrix}$

$= 1000(2(12-4) - 3(9-8) + 4(6-16))$

$|P| = 1000[16 - 3 - 40] = 1000[-27] = -27000$

$\because$ A^{-1} exists, so equations have a unique solution.

$\therefore$ we can find the cost of carry bag using matrices.

(iii) Shopkeeper A is better for environmental condition. As he is using least no. of polythene. Shopkeeper B is better for social conditions as he is using handmade bags.

14. Taking log of both sides, we get

$$\log y = \frac{1}{2}\log(1-x^2) + \frac{1}{2}\log(2x-3) - \frac{2}{3}\log(x^2+2).$$

On differentiating both sides w.r.t.x, we get

$$\frac{1}{y}\frac{dy}{dx} = \frac{1}{2(1-x^2)}(-2x) + \frac{1}{2(2x-3)}\times 2 - \frac{2}{3}\times\frac{1}{x^2+2}2x$$

$$\therefore \quad \frac{dy}{dx} = y\left\{-\frac{x}{1-x^2} + \frac{1}{2x-3} - \frac{4x}{3(x^2+2)}\right\}$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{\sqrt{1-x^2}(2x-3)^{1/2}}{(x^2+2)^{2/3}}\left\{-\frac{x}{1-x^2} + \frac{1}{2x-3} - \frac{4x}{3(x^2+2)}\right\}.$$

15. We have,

$$\cos\frac{x}{2}\cdot\cos\frac{x}{4}\cdot\cos\frac{x}{8}\ldots = \frac{\sin x}{x}$$

Taking log on both sides, we get

$$\log\cos\frac{x}{2} + \log\cos\frac{x}{4} + \log\cos\frac{x}{8}\ldots = \log\sin x - \log x$$

Differentiating both sides with respect to x, we get

$$-\frac{1}{2}\frac{\sin\frac{x}{2}}{\cos\frac{x}{2}} - \frac{1}{4}\frac{\sin\frac{x}{4}}{\cos\frac{x}{4}} - \frac{1}{8}\frac{\sin\frac{x}{8}}{\cos\frac{x}{8}}\ldots = \frac{\cos x}{\sin x} - \frac{1}{x}$$

$$\Rightarrow \quad -\frac{1}{2}\tan\frac{x}{2} - \frac{1}{4}\tan\frac{x}{4} - \frac{1}{8}\tan\frac{x}{8}\ldots = \cot x - \frac{1}{x}$$

Differentiating both sides with respect to x, we get

$$-\frac{1}{2^2}\sec^2\frac{x}{2} - \frac{1}{4^2}\sec^2\frac{x}{4} - \frac{1}{8^2}\sec^2\frac{x}{8}\ldots = \operatorname{cosec}^2 x + \frac{1}{x^2}$$

$$\Rightarrow \quad \frac{1}{2^2}\sec^2\frac{x}{2} + \frac{1}{4^2}\sec^2\frac{x}{4} + \frac{1}{8^2}\sec^2\frac{x}{8}\ldots = \operatorname{cosec}^2 x - \frac{1}{x^2}$$

16. For $f(x)$ to be continuous at $x = 0$, we must have

$$\lim_{x\to 0^-} f(x) = \lim_{x\to 0^+} f(x) = f(0)$$

$$\Rightarrow \quad \lim_{x\to 0^-} f(x) = \lim_{x\to 0^+} f(x) = a \qquad \ldots(i)$$

Now,

$$\lim_{x\to 0^-} f(x) = \lim_{x\to 0}\frac{1-\cos 4x}{x^2}$$

$$\Rightarrow \quad \lim_{x\to 0^-} f(x) = \lim_{x\to 0}\frac{2\sin^2 2x}{x^2}$$

$$\Rightarrow \quad \lim_{x\to 0^-} f(x) = 2\lim_{x\to 0}\left(\frac{\sin 2x}{x}\right)^2$$

$$\Rightarrow \lim_{x\to 0^-} f(x) = 2\times 4\cdot \lim_{x\to 0}\left(\frac{\sin 2x}{2x}\right)^2 = 8(1)^2 = 8 \quad ...(ii)$$

$$\Rightarrow \lim_{x\to 0^+} f(x) = \lim_{x\to 0}\frac{\sqrt{x}}{\sqrt{16+\sqrt{x}}-4}$$

$$\Rightarrow \lim_{x\to 0^+} f(x) = \lim_{x\to 0}\frac{\sqrt{x}}{16+\sqrt{x}-16}\cdot(\sqrt{16+\sqrt{x}}+4)$$

$$\Rightarrow \lim_{x\to 0^+} f(x) = \lim_{x\to 0}\sqrt{16+\sqrt{x}}+4 = 4+4 = 8 \quad ...(iii)$$

From (i), (ii) and (iii), we get $a = 8$.

OR

Differentiating both sides of the given relation w.r.t. x, we get

$$\frac{d}{dx}[\log(x^2+y^2)] = 2\frac{d}{dx}\left\{\tan^{-1}\left(\frac{y}{x}\right)\right\}$$

$$\Rightarrow \frac{1}{x^2+y^2}\times\frac{d}{dx}(x^2+y^2) = 2\times\frac{1}{1+(y/x)^2}\times\frac{d}{dx}\left(\frac{y}{x}\right)$$

$$\Rightarrow \frac{1}{x^2+y^2}\left\{\frac{d}{dx}(x^2)+\frac{d}{dx}(y^2)\right\} = 2\times\frac{x^2}{x^2+y^2}\left\{\frac{x\frac{dy}{dx}-y\times 1}{x^2}\right\}$$

$$\Rightarrow \frac{1}{x^2+y^2}\left\{2x+2y\frac{dy}{dx}\right\} = \frac{2}{x^2+y^2}\left\{x\frac{dy}{dx}-y\right\}$$

$$\Rightarrow 2\left\{x+y\frac{dy}{dx}\right\} = 2\left\{x\frac{dy}{dx}-y\right\} \Rightarrow x+y\frac{dy}{dx} = x\frac{dy}{dx}-y$$

$$\Rightarrow \frac{dy}{dx}(y-x) = -(x+y) \Rightarrow \frac{dy}{dx} = \frac{x+y}{x-y}$$

17. Let $I = \int \tan^{-1}(\sec x+\tan x)dx$.

Then, $I = \int \tan^{-1}\left(\frac{1+\sin x}{\cos x}\right)dx$

$$\Rightarrow I = \int \tan^{-1}\left\{\frac{1-\cos\left(\frac{\pi}{2}+x\right)}{\sin\left(\frac{\pi}{2}+x\right)}\right\}dx$$

$$\Rightarrow I = \int \tan^{-1}\left\{\frac{2\sin^2\left(\frac{\pi}{4}+\frac{x}{2}\right)}{2\sin\left(\frac{\pi}{4}+\frac{x}{2}\right)\cos\left(\frac{\pi}{4}+\frac{x}{2}\right)}\right\}dx$$

$$\Rightarrow I = \int \tan^{-1}\left\{\tan\left(\frac{\pi}{4}+\frac{x}{2}\right)\right\}dx$$

$$\Rightarrow I = \int\frac{\pi}{4}+\frac{x}{2}dx = \frac{\pi}{4}\int 1\cdot dx+\frac{1}{2}x\,dx = \frac{\pi}{4}x+\frac{x^2}{4}+C$$

OR

Let,

$$I = \int\left\{(2x-3)^5+\frac{1}{(7x-5)^3}+\frac{1}{\sqrt{5x-4}}+\frac{1}{2-3x}+\sqrt{3x+2}\right\}dx.$$

Then,

$$I = \int(2x-3)^5dx+\int(7x-5)^{-3}dx+\int(5x-4)^{-1/2}dx$$

$$+\int\frac{1}{2-3x}dx+\int\sqrt{3x+2}\,dx$$

$$\Rightarrow I = \frac{(2x-3)^6}{2\times 6}+\frac{(7x-5)^{-2}}{7\times(-2)}+\frac{(5x-4)^{1/2}}{5\times\frac{1}{2}}+\left(\frac{1}{-3}\right)\log|2-3x|+\frac{(3x+2)^{3/2}}{3\times\frac{3}{2}}+C$$

$$\Rightarrow I = \frac{1}{12}(2x-3)^6-\frac{1}{14}(7x-5)^{-2}+\frac{2}{5}\sqrt{5x-4}-\frac{1}{3}\log|2-3x|+\frac{2}{9}(3x+2)^{3/2}+C$$

18. Let $I = \int\frac{1}{\sqrt[4]{(x-1)^3(x+2)^5}}dx.$

Then, $I = \int\frac{1}{\sqrt[4]{\left(\frac{x-1}{x+2}\right)^3}(x+2)^8}dx$

$$\Rightarrow I = \int\frac{1}{\left(\frac{x-1}{x+2}\right)^{3/4}(x+2)^2}dx$$

$$\Rightarrow I = \int\left(\frac{x-1}{x+2}\right)^{-3/4}\times\frac{1}{(x+2)^2}dx$$

$$d\left(1-\frac{3}{x+2}\right) = dt \Rightarrow \frac{3}{(x+2)^2}dx = dt \Rightarrow \frac{1}{(x+2)^2}dx = \frac{1}{3}dt$$

$$\therefore \quad I = \frac{1}{3}\int t^{-3/4}dt = \frac{4}{3}t^{1/4}+C = \frac{4}{3}\left(\frac{x-1}{x+1}\right)^{1/4}+C$$

19. $I=\int_0^{\pi}\frac{x}{1-\cos\alpha\sin x}dx.$

Then, $I=\int_0^{\pi}\frac{(\pi-x)}{1-\cos\alpha\sin(\pi-x)}dx$

$\Rightarrow \quad I=\int_0^{\pi}\frac{\pi}{1-\cos\alpha\sin x}dx-\int_0^{\pi}\frac{x}{1-\cos\alpha\sin x}dx$

$\Rightarrow \quad I=\pi\int_0^{\pi}\frac{1}{1-\cos\alpha\sin x}dx-I$

$\Rightarrow \quad 2I=\pi\int_0^{\pi}\frac{1}{1-\cos\alpha\sin x}dx$

$\Rightarrow \quad 2I=\pi\int_0^{\pi}\frac{1+\tan^2(x/2)}{1+\tan^2(x/2))-2\cos\alpha\tan(x/2)}dx$

$\Rightarrow \quad 2I=\pi\int_0^{\pi}\frac{\sec^2(x/2)}{\tan^2(x/2)-2\cos\alpha\tan(x/2)+1}dx$

$\Rightarrow \quad I=\frac{\pi}{2}\int_0^{\pi}\frac{\sec^2(x/2)}{\tan^2(x/2)-2\cos\alpha\tan(x/2)+1}dx$

Let $\tan\frac{x}{2}=t$. Then, $d\left(\tan\frac{x}{2}\right)=dt\Rightarrow\sec^2\frac{x}{2}dx=2dt.$

Also, $x=0\Rightarrow t=\tan 0=0$ and $x=\pi\Rightarrow \quad t=\tan\frac{\pi}{2}=\infty.$

$\therefore \quad I=\frac{\pi}{2}\int_0^{\infty}\frac{2dt}{t^2-2t\cos\alpha+1}$

$\Rightarrow \quad I=\pi\int_0^{\infty}\frac{1}{(t-\cos\alpha)^2+(1-\cos^2\alpha)}dt$

$\Rightarrow \quad I=\pi\int_0^{\infty}\frac{1}{\sin^2\alpha+(t-\cos\alpha)^2}dt$

$\Rightarrow \quad I=\frac{\pi}{\sin\alpha}\left[\tan^{-1}\left(\frac{t-\cos\alpha}{\sin\alpha}\right)\right]_0^{\infty}$

$\Rightarrow \quad I=\frac{\pi}{\sin\alpha}\left[\tan^{-1}\infty-\tan^{-1}(-\cot\alpha)\right]$

$\Rightarrow \quad I=\frac{\pi}{\sin\alpha}\left[\frac{\pi}{2}+\tan^{-1}(\cot\alpha)\right]$

$\Rightarrow \quad I=\frac{\pi}{\sin\alpha}\left[\frac{\pi}{2}+\tan^{-1}\left\{\tan\left(\frac{\pi}{2}-\alpha\right)\right\}\right]$

$\Rightarrow \quad I=\frac{\pi}{\sin\alpha}\left[\frac{\pi}{2}+\frac{\pi}{2}-\alpha\right]=\frac{\pi(\pi-\alpha)}{\sin\alpha}$

20. We have, $3\vec{\alpha}=2\vec{\beta}$

$\Rightarrow \quad 3\{(x+4y)\vec{a}+(2x+y+1)\vec{b}\}$

$=2\{(-2x+y+2)\vec{a}+(2x-3y-1)\vec{b}\}$

$\Rightarrow \quad (3x+12y+4x-2y-4)\vec{a}$

$+(6x+3y+3-4x+6y+2)\vec{b}=\vec{0}$

$\Rightarrow \quad 7x+10y-4=0$ and $2x+9y+5=0$

$\Rightarrow \quad x=2,\ y=-1.$

21. We know that the lines

$\frac{x-x_1}{l_1}=\frac{y-y_1}{m_1}=\frac{z-z_1}{n_1}$ and $\frac{x-x_2}{l_2}=\frac{y-y_2}{m_2}=\frac{y-z_2}{n_2}$

are coplanar, if

$$\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2z_1 \\ l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \end{vmatrix}=0$$

The equations of the given lines are

$\frac{x-(a-d)}{\alpha-\delta}=\frac{y-a}{\alpha}=\frac{z-(a+d)}{\alpha+\delta}$ and,

$\frac{x-(b-c)}{\beta-\gamma}=\frac{y-b}{\beta}=\frac{z-(b+c)}{\beta+\gamma}$

Here, $x_1=a-d,\ y_1=a,\ z_1=a+d$

$x_2=b-c,\ y_2=b,\ z_2=b+c$

$l_1=\alpha-\delta,\ m_1=\alpha,\ n_1=\alpha+\delta;\ l_2=\beta-\gamma,\ m_2=\beta,$

$n_2=\beta+\gamma$

$\therefore \quad \begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \end{vmatrix}$

$$=\begin{vmatrix} b-c-a+d & b-a & b+c-a-d \\ \alpha-\delta & \alpha & \alpha+\delta \\ \beta-\gamma & \beta & \beta+\gamma \end{vmatrix}$$

Applying $C_1\to C_1+C_3$

$$=\begin{vmatrix} 2(b-a) & b-a & b+c-a-d \\ 2\alpha & \alpha & \alpha+\delta \\ 2\beta & \beta & \beta+\gamma \end{vmatrix}=0$$

[$\because$ C_1 and C_2 are proportional]

Hence, given lines are coplanar.

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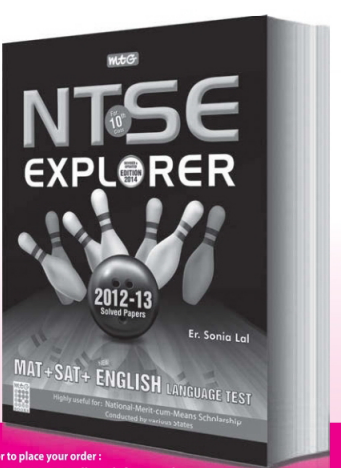

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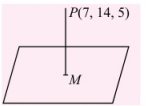
OR

Let M be the foot of the perpendicular from P on the plane $2x + 4y - z = 2$. Then, PM is normal to the plane. So, its direction ratios are 2, 4, –1. Since PM passes through P (7, 14, 5). Therefore, its equation is



$$\frac{x-7}{2} = \frac{y-14}{4} = \frac{z-5}{-1} = r(\text{say})$$

Let the coordinates of M be $(2r + 7, 4r + 14, -r + 5)$.
Since M lies on the plane $2x + 4y - z = 2$. Therefore,

$2(2r + 7) + 4(4r + 14) - (-r + 5) = 2$

$\Rightarrow \quad 21r + 63 = 0 \Rightarrow r = -3$

So, the coordinates of M are (1, 2, 8)

Now,

PM = Length of the perpendicular from P

$$\Rightarrow \quad PM = \sqrt{(7-1)^2 + (14-2)^2 + (5-8)^2} = 3\sqrt{21}.$$

22. As the coin is biased, so according to the given condition $P(\text{Head appears}) = \frac{3}{4}$ and $P(\text{Tail appears}) = \frac{1}{4}$

The random variable X is defined on the sample space given by $S = \{TT, TH, HT, HH\}$
Since X denotes the number of tails, therefore it can take values 0, 1, 2

$$P(X = 0) = P(\text{No tail appears}) = P(HH) = \frac{3}{4} \times \frac{3}{4} = \frac{9}{16}$$

$P(X = 1) = P(\text{one tail appears})$

$$= P(TH) + P(HT) = 2\left[\frac{1}{4} \times \frac{3}{4}\right] = \frac{6}{16}$$

$$P(X = 2) = P(\text{Two tail appears}) = P(TT) = \frac{1}{4} \times \frac{1}{4} = \frac{1}{16}$$

$X = x$	0	1	2
$P(X = x)$	$\frac{9}{16}$	$\frac{6}{16}$	$\frac{1}{16}$

$$\text{Mean} = \sum_{i=1}^{3} p_i x_i = 0 \times \frac{9}{16} + 1 \times \frac{6}{16} + 2 \times \frac{1}{16} = \frac{1}{2}$$

No, it may be good once or twice but not forever.
Honesty pays in a long run.

23. We have, $|A| = \begin{vmatrix} 1 & \tan x \\ -\tan x & 1 \end{vmatrix} = 1 + \tan^2 x \neq 0$

So, A is invertible. Let C_{ij} be the cofactor of a_{ij} in $A = [a_{ij}]$.

$C_{11} = (-1)^{1+1} 1 = 1$, $C_{12} = (-1)^{1+2}(-\tan x) = \tan x$

$C_{21} = (-1)^{2+1} \tan x = -\tan x$ and $C_{22} = (-1)^{2+2} 1 = 1$

$$\therefore \quad \text{adj}A = \begin{bmatrix} 1 & \tan x \\ -\tan x & 1 \end{bmatrix}^T = \begin{bmatrix} 1 & -\tan x \\ \tan x & 1 \end{bmatrix}.$$

Now, $A^{-1} = \frac{1}{|A|}\text{adj}A$

$$\Rightarrow \quad A^{-1} = \frac{1}{1+\tan^2 x}\begin{bmatrix} 1 & -\tan x \\ \tan x & 1 \end{bmatrix}$$

$$\Rightarrow \quad A^{-1} = \begin{bmatrix} \frac{1}{1+\tan^2 x} & \frac{-\tan x}{1+\tan^2 x} \\ \frac{\tan x}{1+\tan^2 x} & \frac{1}{1+\tan^2 x} \end{bmatrix}$$

$$\therefore \quad A^T A^{-1} = \begin{bmatrix} 1 & -\tan x \\ \tan x & 1 \end{bmatrix}\begin{bmatrix} \frac{1}{1+\tan^2 x} & \frac{-\tan x}{1+\tan^2 x} \\ \frac{\tan x}{1+\tan^2 x} & \frac{1}{1+\tan^2 x} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{1+\tan^2 x} - \frac{\tan^2 x}{1+\tan^2 x} & -\frac{\tan x}{1+\tan^2 x} - \frac{\tan x}{1+\tan^2 x} \\ \frac{\tan x}{1+\tan^2 x} + \frac{\tan x}{1+\tan^2 x} & -\frac{\tan^2 x}{1+\tan^2 x} + \frac{1}{1+\tan^2 x} \end{bmatrix}$$

$$\Rightarrow \quad A^T A^{-1} = \begin{bmatrix} \frac{1-\tan^2 x}{1+\tan^2 x} & \frac{-2\tan x}{1+\tan^2 x} \\ \frac{2\tan x}{1+\tan^2 x} & \frac{1-\tan^2 x}{1+\tan^2 x} \end{bmatrix}$$

$$\Rightarrow \quad A^T A^{-1} = \begin{bmatrix} \cos 2x & -\sin 2x \\ \sin 2x & \cos 2x \end{bmatrix}.$$

24. Let the required point be (x_1, y_1).
The equation of the curve is $9y^2 = x^3$.
Since (x_1, y_1) lies on the curve. Therefore,

$$9y_1^2 = x_1^3 \qquad \ldots(i)$$

Now, $9y^2 = x^3$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{x^2}{6y} \quad \Rightarrow \quad \left(\frac{dy}{dx}\right)_{(x_1, y_1)} = \frac{x_1^2}{6y_1}.$$

Since the normal to the curve at (x_1, y_1) make equal intercepts with the coordinate axes.
Therefore, Slope of the normal = ±1

$\Rightarrow \quad -\dfrac{1}{\left(\dfrac{dy}{dx}\right)_{(x_1,y_1)}} = \pm 1 \quad \Rightarrow \quad \left(\dfrac{dy}{dx}\right)_{(x_1,y_1)} = \pm 1$

$\Rightarrow \quad \dfrac{x_1^2}{6y_1} = \pm 1 \qquad \Rightarrow \quad x_1^2 = \pm\, 6y_1 \Rightarrow\ x_1^4 = 36y_1^2$

$\Rightarrow \quad x_1^4 = 36\left(\dfrac{x_1^3}{9}\right)$ [Using (i)]

$\Rightarrow \quad x_1^4 = 4x_1^3 \Rightarrow\ x_1^3(x_1 - 4) = 0$

$\Rightarrow \quad x_1 = 0, 4$

Putting $x_1 = 0$ in (i), we get $9y_1^2 = 0 \Rightarrow y_1 = 0$

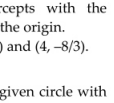
Putting $x_1 = 4$ in (i), we get $9y_1^2 = 4^3 \Rightarrow y_1 = \pm\dfrac{8}{3}$

But, the line making equal intercepts with the coordinate axes cannot pass through the origin.
Hence, the required points are (4, 8/3) and (4, –8/3).

OR

Let ABC be a triangle inscribed in a given circle with centre O and radius r. The area of the triangle will be maximum if its vertex A opposite to the base BC is at a maximum distance from the base BC. This is possible only when A lies on the diameter perpendicular to BC. Thus, $AD \perp BC$. So, triangle ABC must be an isosceles triangle.

Let $OD = x$.



In right triangle ODB, we have

$OB^2 = OD^2 + BD^2$

$\Rightarrow \quad r^2 = x^2 + BD^2 \Rightarrow \quad BD = \sqrt{r^2 - x^2}$

$\Rightarrow \quad BC = 2BD = 2\sqrt{r^2 - x^2}$.

Also, $AD = AO + OD = r + x$.

Let A denote the area of ΔABC. Then,

$A = \dfrac{1}{2}(BC \times AD)$

$\Rightarrow \quad A = \dfrac{1}{2} \times 2\sqrt{r^2 - x^2} \times (r + x)$

$\Rightarrow \quad A = (r + x)\sqrt{r^2 - x^2}$

$\Rightarrow \quad \dfrac{dA}{dx} = \sqrt{r^2 - x^2} - \dfrac{x(r + x)}{\sqrt{r^2 - x^2}}$

$\Rightarrow \quad \dfrac{dA}{dx} = \dfrac{r^2 - rx - 2x^2}{\sqrt{r^2 - x^2}}$

For maximum or minimum values of A, we must have $\dfrac{dA}{dx} = 0$

$\Rightarrow \quad \dfrac{r^2 - rx - 2x^2}{\sqrt{r^2 - x^2}} = 0$

$\Rightarrow \quad (r - 2x)(r + x) = 0 \Rightarrow r - 2x = 0 \qquad [\because \ \ r + x \neq 0]$

$\Rightarrow \quad x = \dfrac{r}{2}$

Now, $\dfrac{dA}{dx} = \dfrac{r^2 - rx - 2x^2}{\sqrt{r^2 - x^2}}$

$\Rightarrow \quad \dfrac{d^2A}{dx^2} = \dfrac{(-r - 4x)}{\sqrt{r^2 - x^2}} + \dfrac{(r^2 - rx - 2x^2)x}{(r^2 - x^2)^{3/2}}$

$\Rightarrow \quad \left(\dfrac{d^2A}{dx^2}\right)_{x = r/2} = -2\sqrt{3} < 0$

Thus, A is maximum when $x = \dfrac{r}{2}$

$\therefore \quad BD = \sqrt{r^2 - x^2} \Rightarrow BD = \dfrac{\sqrt{3}r}{2}$

In ΔODB, Let $\angle BOD = \theta$

$\tan\theta = \dfrac{BD}{OD} \Rightarrow \tan\theta = \dfrac{\frac{\sqrt{3}r}{2}}{\frac{r}{2}} = \sqrt{3} \Rightarrow \theta = 60°$

$\therefore \quad \angle BAC = \theta = 60°$

But $AB = AC \quad \therefore \quad \angle B = \angle C$

$\Rightarrow \quad \angle A = \angle B = \angle C = 60°$

$\Rightarrow \quad A$ is maximum when ΔABC is equilateral.

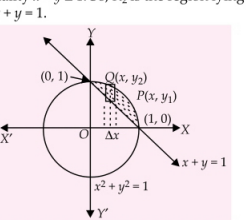
25. Let $R = \{(x, y) : x^2 + y^2 \leq 1 \leq x + y\}$. Then

$\Rightarrow \quad R = R_1 \cap R_2$, where

$R_1 = \{(x, y) : x^2 + y^2 \leq 1\}$ and $R_2 = \{(x, y) : 1 \leq x + y\}$

R_1 : Clearly $x^2 + y^2 = 1$ represents a circle with centre at (0, 0) and radius 1.
Since $x^2 + y^2 \leq 1$, so region R_1 represents the interior of circle $x^2 + y^2 = 1$.

R_2 : Since $x + y = 1$ is the equation of a straight line cutting x and y-axes at (1, 0) and (0, 1) respectively. This line divides the xy-plane in two parts represented by $x + y \leq 1$ and $x + y \geq 1$. Since (0, 0) does not satisfy the inequality $x + y \geq 1$. So, R_2 is the region lying above the line $x + y = 1$.



Hence, region R is the shaded region shown in figure. When we slice the shaded region into vertical strips, we observe that each vertical strip has its lower end on the line $x + y = 1$ and the upper end on $x^2 + y^2 = 1$. So, the approximating rectangle shown in fig. has, Length $= (y_2 - y_1)$, Width $= \Delta x$ and Area $= (y_2 - y_1)\Delta x$. Since the approximating rectangle can move from $x = 0$ to $x = 1$. So, required area is given by

$$A = \int_0^1 (y_2 - y_1)dx \quad \Rightarrow \quad A = \int_0^1 \left\{\sqrt{1-x^2} - (1-x)\right\}dx$$

$$\Rightarrow \quad A = \left[\frac{1}{2}x\sqrt{1-x^2} + \frac{1}{2}\sin^{-1}\left(\frac{x}{1}\right) - x + \frac{x^2}{2}\right]_0^1$$

$$\Rightarrow \quad A = \left\{0 + \frac{1}{2}\sin^{-1}1 - 1 + \frac{1}{2}\right\} - 0 = \left(\frac{\pi}{4} - \frac{1}{2}\right) \text{sq.units}$$

26. $(xe^{y/x} + y)dx = xdy$

$$\Rightarrow \quad \frac{dy}{dx} = e^{y/x} + \frac{y}{x}$$

This is a homogeneous differential equation. Putting $y = vx$ and $\frac{dy}{dx} = v + x\frac{dv}{dx}$ reduces to

$$v + x\frac{dv}{dx} = e^v + v$$

$$\Rightarrow \quad x\frac{dv}{dx} = e^v \;\Rightarrow\; e^{-v}dv = \frac{dx}{x}, \text{ if } x \neq 0.$$

$$\Rightarrow \quad \int e^{-v}dv = \int \frac{1}{x}dx$$

$$\Rightarrow \quad -e^{-v} = \log|x| + C$$

$$\Rightarrow \quad -e^{-y/x} = \log|x| + C \qquad \ldots(i)$$

It is given that $y(1) = 1$ *i.e.* when $x = 1$, $y = 1$.

Putting $x = 1$, $y = 1$ in (i), we get $-e^{-1} = C$

Putting $C = -\frac{1}{e}$ in (i), we get

$$-e^{-y/x} = \log|x| - \frac{1}{e}$$

$$\Rightarrow \quad e^{-y/x} = \frac{1}{e} - \log|x|$$

$$\Rightarrow \quad -\frac{y}{x} = \log(1 - e\log|x|) - 1$$

$$\Rightarrow \quad y = x - x\log(1 - e\log|x|)$$

Hence, $y = x - x\log(1 - e\log|x|)$, is the solution of the given equation.

27. We have,

$$\frac{x-1}{3} = \frac{y+1}{2} = \frac{z-1}{5} = \lambda \text{ (say)}$$

$$\Rightarrow \quad x = 3\lambda + 1,\ y = 2\lambda - 1,\ z = 5\lambda + 1.$$

So, the coordinates of a general point on this line are $(3\lambda + 1, 2\lambda - 1, 5\lambda + 1)$.

The equation of the second line is

$$\frac{x+2}{4} = \frac{y-1}{3} = \frac{z+1}{-2} = \mu \text{ (say)}$$

$$\Rightarrow \quad x = 4\mu - 2,\ y = 3\mu + 1,\ z = -2\mu - 1$$

So, the coordinates of a general point on this line are $(4\mu - 2, 3\mu + 1, -2\mu - 1)$.

If the line intersect, then they have a common point. So, for some values of λ and μ, we must have

$$3\lambda + 1 = 4\mu - 2,\ 2\lambda - 1 = 3\mu + 1 \text{ and } 5\lambda + 1 = -2\mu - 1.$$

$$\Rightarrow \quad 3\lambda - 4\mu = -3 \qquad \ldots(i)$$

$$2\lambda - 3\mu = 2 \qquad \ldots(ii)$$

and, $5\lambda + 2\mu = -2 \qquad \ldots(iii)$

Solving (i) and (ii), we obtain $\lambda = -17$ and $\mu = -12$.

These values of λ and μ do not satisfy the third equation.

Hence, the given lines do not intersect.

28. Let the dietician mix x kg of food I and y kg of food II.

Since 1 kg of food I cost ₹ 5 and 1 kg of food II cost ₹ 7.

$\therefore$ Total cost of x kg of food I and y kg of food II is ₹ $(5x + 7y)$

Minimize $z = 5x + 7y$

The given data may be put in the following tabular form:

Vitamin	Food		Requirement
	I	II	
A	2	1	8
C	1	2	10
Cost (₹)	5	7	

Since the minimum requirement of vitamin A and C in food I respectiely is

$2x + 1y \geq 8 \qquad \ldots(i)$

Since the minimum requirement of vitamin A and C in food II respectively is

$x + 2y \geq 10 \qquad \ldots(ii)$

Hence, the mathematical model of the LPP is as follows:

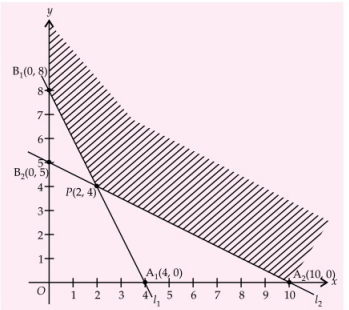
Minimize $z = 5x + 7y$

subject to $2x + 1y \geq 8$

$x + 2y \geq 10$ and $x, y \geq 0$

To solve this LPP graphically, we first convert the inequations into equations to obtain the following lines

$l_1 : 2x + y = 8,\ l_2 : x + 2y = 10,\ l_3 : x = 0,\ l_4 = y = 0$



The line l_1 meets the co-ordinate axes at $A_1(4, 0)$ and $B_1(0, 8)$.

The line l_2 meets the co-ordinates axes at $A_2(10, 0)$ and $B_2(0, 5)$

Clearly $x \geq 0$, $y \geq 0$ represents the first quadrant.

The shaded region in figure is the feasible region of the LPP.

The coordinates of the corner points of this region are $A_2(10, 0)$, $P(2, 4)$, $B_1(0, 8)$.

The point $P(2, 4)$ is obtained by solving $2x + y = 8$ and $x + 2y = 10$ simultaneously.

The values of the objective function $z = 5x + 7y$ at the corner points of the feasible region are given in the following table.

Point (x, y)	**Value of obj. function $z = 5x + 7y$**
$A_2(10, 0)$	$z = 50$
$P(2, 4)$	$z = 38$
$B_1(0, 8)$	$z = 56$

Minimum cost = ₹ 38.00

$x = 2$, $y = 4$

Balanced diet keeps fit, healthy and gives disease free life to a person.

29. Let x denote the number of non-vegetarian selected at random.

Hence, x can take values 0, 1, 2.

$P(x = 0) = P$(No non-vegetarian people is selected)

$$= \frac{13}{20} \times \frac{12}{19} = \frac{156}{380}$$

$P(x = 1) = P$(one is vegetarian and second is non-vegetarian)

$$= \frac{{}^{13}C_1 \times {}^{7}C_1}{{}^{20}C_2}$$

$$= \frac{13 \times 7}{\left(\frac{20 \times 19}{2}\right)} = \frac{182}{380}$$

$P(x = 2) = $ P(Both are non-vegetarian)

$$= \frac{7}{20} \times \frac{6}{19} = \frac{42}{380}$$

x	0	1	2
$p(x)$	$\frac{156}{380}$	$\frac{182}{380}$	$\frac{42}{380}$

I would like to be a vegetarian because vegetarian food is much easier to digest than non vegetarian (may be given other reason) Or for non-vegetarian food we have to kill animals this is not good thing because everybody has right to survive, etc.

OR

Since the chairman is biased, so the probability that he selects his relatives for a job is 3 times as likely as others.

P(He selects his relative) = $\frac{3}{4}$

P(He does not select his relative) = $\frac{1}{4}$

$P(x = 0) = P$(All three are his relatives)

$$= \frac{3}{4} \times \frac{3}{4} \times \frac{3}{4} = \frac{27}{64}$$

$P(x = 1) = P$(one of them is other than the relative)

$$= \left(\frac{1}{4} \times \frac{3}{4} \times \frac{3}{4}\right) = \frac{9}{64}$$

$P(x = 2) = P$(Two of them are other than the relatives)

$$= \left(\frac{1}{4} \times \frac{3}{4} \times \frac{1}{4}\right) = \frac{3}{64}$$

$P(x = 3) = P$(All three are other than relatives)

$$= \frac{1}{4} \times \frac{1}{4} \times \frac{1}{4} = \frac{1}{64}$$

x	0	1	2	3
$P(x)$	$\frac{27}{64}$	$\frac{9}{64}$	$\frac{3}{64}$	$\frac{1}{64}$

Values lost by chairman Honesty and Integrity

MATHS MUSING

SOLUTION SET-133

1. (c) : $\frac{1}{a}+\frac{1}{b}=\frac{1}{x}\rightarrow$

$(a-x)(b-x)=x^2$.

Number of pairs (a, b) is the number of divisors of $x^2=2^2.19^2.53^2$.

The number of pairs (a, b), $a<b$ is

$$\frac{3\cdot3\cdot3-1}{2}=13$$

2. (c) : The points on the lines are $P(-2+2s, -6+3s, 34-10s)$, $Q(-6+4t, 7-3t, 7-2t)$

PQ is perpendicular to the two lines.

$\therefore \quad s=3,\ t=2,\ P=(4, 3, 4),\ a+b+c=11$

3. (b) : $|\vec{a}\times\vec{b}|^2=12,\ \vec{c}=2\vec{a}\times\vec{b}-3\vec{b}$

$|\vec{c}|^2=4|\vec{a}\times\vec{b}|^2+9|\vec{b}|^2=192$

$\angle(\vec{b},\vec{c})=\theta,\ \cos\theta=-\frac{\sqrt{3}}{2},\ \therefore\ \theta=\frac{5\pi}{6}$

4. (b) : $a^2p+b^2q=a^3+b^3,\ p^3+q^3=a^3+b^3$

$p=a\alpha,\ q=b\beta\ \rightarrow(\alpha-1)a^3+(\beta-1)b^3=0$ and

$(\alpha^3-1)a^3+(\beta^3-1)b^3=0\rightarrow\alpha+\beta+1=0$

$\therefore \quad bp+aq+ab=0$

5. (b) : The number of 5-element subsets = $\binom{15}{5}=3003$. The number of subsets without consecutive numbers = $\binom{15-5+1}{5}\binom{11}{5}=462$.

The desired number of subsets = 3003 – 462 = 2541, with digit sum 12.

6. (a, b, c, d)

$\frac{2ab}{a+b}=2014\rightarrow(a-1007)(b-1007)=1007^2$

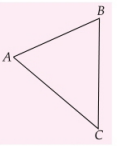
$=19^2.53^2$

$\therefore \quad a=1007+1=1008,\ a=1007+69=1026,$

$a=1007+53=1060,\ a=1007+19^2=1368$

7. (c) : $A=(t_1^2,2t_1),B=(t_2^2,2t_2)$

$C=(t^2,2t),t_1t_2=4,\ t_1+t_2+t=0$



Altitude from A is $t_1x-2y=t_1^3-4t_1$

Altitude from B is $t_2x-2y=t_2^3-4t_2$

Solving, we get orthocenter $O(t^2-6, -t)$

Locus of O is $y^2=x+6$

8. (a) : $G=\left(\frac{2}{3}(t^2-2),0\right)$ divides OS in the ratio

$2:1, \quad \therefore \quad S=\left(1+\frac{t^2}{2},\frac{t}{2}\right)$

The locus of circumcentre S is $y^2=\frac{1}{2}(x-1)$

9. (9) : $c^2=a^2+b^2\rightarrow x=\frac{ab}{a+b+\sqrt{a^2+b^2}}$

$\rightarrow \quad (a-2r)(b-2r)=2r^2=2^3.19^2.53^2$

$x=\frac{1}{2}\times$ number of divisors of $2r^2=18$

10. (b) :

P. $S=1+2-3-4+\ldots.-2012+2013+2014$

$=-2012+2013+2014=2015$

Q. $S=1+3-5-7+\ldots.-4023+4025+4027$

$=-4024+4025+4027=4028$

R. $S=-2012.2013+2013^2+2014^2=2013+2014^2$

$=09$ (Last 2 digits)

S. $S=-4024^2+4025^2+4027^2$

$=8049+4027^2=78$ (Last 2 digits)

■■

Solution Sender of Maths Musing

SET-133

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Math Archives 10 Best Problems

Math Archives, as the title itself suggests, is a collection of various challenging problems related to the topics of IIT-JEE Syllabus. This section is basically aimed at providing an extra insight and knowledge to the candidates preparing for IIT-JEE. In every issue of MT, challenging problems are offered with detailed solution. The readers' comments and suggestions regarding the problems and solutions offered are always welcome.

1. The centre of a square $ABCD$ is at z_0. If A is z_1, then the centroid of the ΔABC is

(a) $2z_0 - i(z_1 - z_0)$ (b) $z_0 + i\left(\frac{z_1 - z_0}{3}\right)$

(c) $\frac{z_0 + iz_1}{3}$ (d) $\frac{2}{3}(z_1 - z_0)$

2. The reflection of the complex number $(2 - i)$ in the straight line $iz = \bar{z}$ is

(a) $4 - 3i$ (b) $3 + 4i$

(c) $2 + i$ (d) $1 - 2i$

3. If sum of the arguments of all the roots of the equation $z^n = k(k$ is real) is odd multiple of π, then it can be true when

(i) n is odd and $k > 0$ (ii) n is odd and $k < 0$

(iii) n is even and $k > 0$ (iv) n is even and $k < 0$

(a) (i) and (iii) are correct

(b) (i) and (iv) are correct

(c) (ii) and (iii) are correct

(d) (ii) and (iv) are correct

4. Let $f : R \to \left(0, \frac{\pi}{2}\right]$ be a function defined by $f(x) = \cot^{-1}(x^2 + 4x + \alpha^2 - \alpha)$, then complete set of values of α for which $f(x)$ is onto, is

(a) $\left[\frac{1-\sqrt{17}}{2}, \frac{1+\sqrt{17}}{2}\right]$

(b) $\frac{1 \pm \sqrt{17}}{2}$

(c) $\left(-\infty, \frac{1-\sqrt{17}}{2}\right] \cup \left[\frac{1+\sqrt{17}}{2}, \infty\right)$

(d) none of these

5. Let $g(x)$ and $f(x)$ be twice differentiable functions in R and $f(2) = 8$, $g(2) = 0$, $f(4) = 10$ and $g(4) = 8$, then

(a) $g'(x) > 4f''(x)\ \forall x \in (2, 4)$

(b) $g(x) > f(x)\ \forall x \in (2, 4)$

(c) $3g'(x) = 4f'(x)$ for at least one $x \in (2, 4)$

(d) $g'(x) = 4f'(x)$ for at least one $x \in (2, 4)$

6. If A, G, H be respectively the AM, GM and the HM between two positive numbers and if $xA = yG = zH$ where x, y, z are non zero positive quantities, then x, y, z are in

(a) AP (b) GP

(c) HP (d) AGP

7. Given $\int_0^{\pi/2} \frac{dx}{1 + \sin x + \cos x} = \ln 2$, then the value of the definite integral $\int_0^{\pi/2} \frac{\sin x}{1 + \sin x + \cos x}\, dx$ is

(a) $\frac{1}{2}\ln 2$ (b) $\frac{\pi}{2} - \ln 2$

(c) $\frac{\pi}{4} - \frac{1}{2}\ln 2$ (d) $\frac{\pi}{2} + \ln 2$

8. Let $y = \{x\}^{[x]}$ where $\{x\}$ denotes the fractional part of x and $[x]$ denotes greatest integer $\le x$, then $\int_0^3 y\,dx =$

(a) $\frac{11}{6}$ (b) $\frac{2}{3}$

(c) 1 (d) $\frac{5}{6}$

9. A quadratic equation $f(x) = ax^2 + bx + c = 0$ $(a \neq 0)$ has positive distinct roots reciprocal of each other. Which one is correct?

(a) $f'(1) = 0$

(b) $af'(1) < 0$

(c) $af'(1) > 0$

(d) nothing can be said about $af'(1)$

By : **Prof. Shyam Bhushan,** Director, Narayana IIT Academy, Jamshedpur. Mob. : 09334870021

10. Each of 10 passengers board any of the three buses randomly, which had no passenger initially. The probability that each bus has got at least one passenger, is

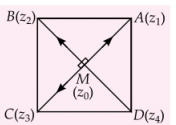
(a) $\frac{{}^{10}p_3\, 3^7}{3^{10}}$ (b) $1-\frac{{}^{10}C_3\, 3^7}{3^{10}}$

(c) $1-\frac{2^{10}}{3^{10}}$ (d) $\frac{3^{10}-3\cdot 2^{10}+3}{3^{10}}$

SOLUTIONS

1. (b) : From the figure along side,

$z_2 - z_0 = i(z_1 - z_0)$



$\Rightarrow \quad z_2 = z_0 + i(z_1 - z_0)$ and $z_3 - z_0 = -(z_1 - z_0)$

$\Rightarrow \quad z_3 = 2z_0 - z_1$

Hence, centroid of ΔABC

$= \frac{z_1 + z_2 + z_3}{3} = z_0 + i\left(\frac{z_1 - z_0}{3}\right)$

2. (d) : $iz = \overline{z}$

$\Rightarrow \quad i(x + iy) = x - iy$

$\Rightarrow \quad i(x + y) - (x + y) = 0$

$\Rightarrow \quad (i - 1)(x + y) = 0$

which represents the line $y = -x$

So, reflection of the point (2, –1) in the line $y = -x$ gives the point (1, –2).

3. (c) : When n is odd, and $k > 0$

Sum of arguments = even multiple of π.

When n is odd and $k < 0$

Sum of arguments = odd multiple of π.

When n is even and $k > 0$

Sum of arguments = odd multiple of π.

When n is even and $k < 0$

Sum of arguments = even multiple of π.

4. (b) : Clearly $x^2 + 4x + \alpha^2 - \alpha \geq 0 \ \forall\ x \in R$ and must take all values of the interval $[0, \infty)$

$\Rightarrow \quad D = 0$

i.e., $16 - 4(\alpha^2 - \alpha) = 0 \ \Rightarrow \ \alpha^2 - \alpha = 4$

$\Rightarrow \quad \alpha = \frac{1 \pm \sqrt{17}}{2}$.

5. (d) : Let $h(x) = g(x) - 4f(x)$

Verify the Rolle's theorem in (2, 4)

Now $h(2) = g(2) - 4f(2) = 0 - 4 \times 8 = -32$

$h(4) = g(4) - 4f(4) = 8 - 40 = -32$

$\Rightarrow \quad h'(x) = 0$ for at least one $x \in (2, 4)$

$\therefore \quad g'(x) = 4f'(x)$ for at least one $x \in (2, 4)$.

6. (b) : Let two numbers be a and b

$x.\frac{a+b}{2} = y.\sqrt{ab} = z.\frac{2ab}{a+b}$

$\Rightarrow \quad \frac{a+b}{2\sqrt{ab}} = \frac{y}{x} = \frac{z}{y}$

$\Rightarrow \quad y^2 = xz \ \Rightarrow \ x, y, z$ are in GP.

7. (c) : $I = \int_0^{\pi/2} \frac{1 + \sin x + \cos x - 1 - \cos x}{1 + \sin x + \cos x}\, dx$

$= \int_0^{\pi/2} dx - \int_0^{\pi/2} \frac{dx}{1 + \sin x + \cos x} - \int_0^{\pi/2} \frac{\cos x}{1 + \sin x + \cos x} dx$

$I = \frac{\pi}{4} - \frac{1}{2}\ln 2$.

8. (a) :

$\int_0^3 (x - [x])^{[x]} dx = \int_0^1 dx + \int_1^2 (x-1)dx + \int_2^3 (x-2)^2 dx$

Put $y = x - 1$, $z = x - 2$

$= 1 + \int_0^1 y dy + \int_0^1 z^2 dz = 1 + \left[\frac{y^2}{2}\right]_0^1 + \left[\frac{z^3}{3}\right]_0^1$

$= 1 + \frac{1}{2} + \frac{1}{3} = \frac{11}{6}$.

9. (b) : $f(x) = ax^2 + bx + c = 0$ $(a \neq 0)$

Let α and $\frac{1}{\alpha}$ be its roots.

$\alpha + \frac{1}{\alpha} = -\frac{b}{a} > 2$

$\Rightarrow \quad \left(\frac{2a+b}{a}\right) < 0 \ \Rightarrow a(2a + b) < 0$

$f'(x) = 2ax + b$

$af'(1) = a(2a + b) < 0$

10. (d) : Total ways in which they can be seated in the buses = 3^{10}

Number of cases in which each bus has got at least one passenger = $3^{10} - 3C_1 \times 1 - 3C_2(2^{10} - 2)$

$\therefore \quad$ Required probability $= \frac{3^{10} - 3C_1 \times 1 - 3C_2\left(2^{10} - 2\right)}{3^{10}}$

$= \frac{3^{10} - 3\cdot 2^{10} + 3}{3^{10}}$.

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The best questions and their solutions will be printed in this column each month.

Q.1 Find the locus of z if

$$(z-1)\left(\cos\frac{\pi}{3}-i\sin\frac{\pi}{3}\right)+\frac{1}{(z-1)}\left(\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}\right)$$

is real.

– Ishaan (Punjab)

Ans. $(x+iy-1)\left(\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)+\frac{1}{(x+iy-1)}\left(\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)$

$$=(x-1+iy)\left(\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)+\frac{(x-1-iy)\left(\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)}{(x-1)^2+y^2}$$

$$=\frac{1}{2}(x-1+\sqrt{3}y)+\frac{i}{2}(y-\sqrt{3}(x-1))$$

$$+\frac{1}{(x-1)^2+y^2}\left[\frac{x-1+\sqrt{3}\,y}{2}-\frac{i}{2}(y-\sqrt{3}(x-1)\right]$$

is real, then $(y-\sqrt{3}(x-1))\left(1-\frac{1}{(x-1)^2+y^2}\right)=0.$

The locus consists of the circle $(x-1)^2+y^2=1$ and its diameter $y=\sqrt{3}(x-1)$

Q.2 If $f(x)=\frac{x}{\sin x}$ and $g(x)=\frac{x}{\tan x}$ where $0<x\le 1$, then prove that in this interval $f(x)$ is increasing and $g(x)$ is decreasing.

– Prateep (Haryana)

Ans. $f(x)=\frac{x}{\sin x}$ and $g(x)=\frac{x}{\tan x}$, where $0<x\le 1$

$$f'(x)=\frac{\sin x-x\cos x}{\sin^2 x}=\frac{p(x)}{\sin^2 x} \quad ...(1)$$

$p(x)=\sin x-x\cos x\Rightarrow p'(x)=\cos x-\cos x+x\sin x$

$\Rightarrow \quad p'(x)>0$ in $(0<x\le 1)$

$\therefore \quad p(x)$ is increasing

But $p(0)=0 \quad \Rightarrow \quad p(x)>0$ for $x>0$

$\therefore$ from (1), $f'(x)>0 \quad \Rightarrow \quad f(x)$ is increasing.

Now, $g(x)=\frac{x}{\tan x}\Rightarrow g'(x)=\frac{\tan x-x\sec^2 x}{\tan^2 x}$

$$\Rightarrow g'(x)=\frac{\sin x\cos x-x}{\sin^2 x}=\frac{\sin 2x-2x}{2\sin^2 x}=\frac{q(x)}{2\sin^2 x}$$

where $q(x)=\sin 2x-2x\Rightarrow q'(x)=2(\cos 2x-1)<0$

$\Rightarrow \quad q(x)$ is decreasing. But $q(0)=0$

$\therefore \quad q(x)<0\Rightarrow g'(x)<0 \Rightarrow g(x)$ is decreasing.

Q.3 Let A be a set containing n elements. Three subsets P, Q and R of A are selected at random. Find the probability that $P\cup Q\cup R=A$.

– Vishnu (Bangalore)

Ans. Let $A=\{a_1, a_2, a_3, ..., a_n\}$. Consider the element a_1. There are the following possibilities on a_1, namely

(1) a_1 may belong to P alone.
(2) a_1 may belong to Q alone.
(3) a_1 may belong to R alone.
(4) a_1 may belong to P and Q both but not to R.
(5) a_1 may belong to P and R both but not to Q.
(6) a_1 may belong to Q and R both but not to P.
(7) a_1 may belong to P, Q and R.
(8) a_1 may not belong to any of P, Q and R.

First 7 cases are in favour of the event that a_1 will belong to $P\cup Q\cup R$. Hence the chance that a_1 will lie in $P\cup Q\cup R$, is 7/8. Similarly, the chance that a_2 will lie in this set, is 7/8 and so on. Therefore, the probability that all the elements of A will be in this set, is $(7/8)^n$.

Q.4 If $f(x)$ is a polynomial of least degree that has a maximum 6 at $x=1$ and a minimum equal to 2 at $x=3$, then find the value of $\int_0^1 f(x)\,dx$

– Sahil Garg (New Delhi)

Ans. $f(x)$ being a polynomial, is differentiable therefore points of maxima and minima are roots of $f'(x)=0$. Since derivative of a polynomial is a polynomial, so the least degree polynomial with roots 1 and 3 is of the form $k(x-1)(x-3)$.

$$\Rightarrow \quad f'(x)=k(x-1)(x-3)$$

$$\Rightarrow \quad f(x)=k\int_1^x (x-1)(x-3)\,dx+c$$

$$\Rightarrow \quad f(x)=k\left(\frac{x^3}{3}-\frac{4x^2}{2}+3x\right)+c \quad ...(1)$$

$$\therefore \ 6=f(1)=k\left(\frac{1}{3}-2+3\right)+c\Rightarrow\frac{4k}{3}+c=6 \quad ...(2)$$

Also $2=f(3)=k(9-18+9)+c \ \Rightarrow \ c=2$ ...(3)

$\therefore$ From (2), $k=3$

$\therefore$ From (1), $f(x)=x^3-6x^2+9x+2$

$$\Rightarrow \quad \int_0^1 f(x)\,dx=\frac{19}{4}.$$

■■

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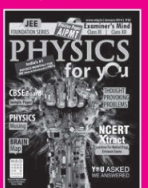

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